Date. 6/9/18

Bisection Method

1 Find the approximate root of the equation $x^3-x-1=0$ by using bi-section method.

Solu) Given

$$f(x) = x^3-x-1$$

$x=0;\ f(0) = 0-0-1 = -1$ -ve

$x=1;\ f(1) = 1-1-1 = -1$ -ve

$x=2;\ f(2) = 2^3-2-1 = 8-3 = 5$ +ve

The root lies between 1 and 2

$$x_0 = \frac{1+2}{2} = \frac{3}{2} = 1.5$$

S.No	a(-ve)	b(+ve)	$x_n = \frac{a+b}{2}$
1	1	2	1.5 (+ve)
2	1	1.5	1.25 (-ve)
3	1.25	1.5	1.375 (+ve)
4	1.25	1.375	1.3125 (-ve)
5	1.3125	1.375	1.3438 (+ve)
6	1.3125	1.3438	1.3282 (+ve)
7	1.3125	1.3282	1.3204 (-ve)
8	1.3204	1.3282	1.3243 (-ve)
9	1.3243	1.3282	1.3263 (+ve)
10	1.3243	1.3263	1.3253 (+ve)
11	1.3243	1.3253	1.3248 (+ve)
12	1.3243	1.3248	1.3246 (-ve)
13	1.3246	1.3248	1.3247 (-ve)
14	1.3247	1.3248	1.3248 (+ve)
15	1.3247	1.3248	1.3248 (

$x_{14} = x_{15} = 1.3248$

2 find the approximate root of the equation $\cos x - xe^x$ by bisection method

Solu) consider

$$f(x) = \cos x - xe^x$$

$x=0$ $f(0) = \cos 0 - 0e^0$

$= 1$ +ve

$x=1$ $f(1) = \cos(1) - 1e^1$

$= 0.540302305 - 2.718281828$

$= -2.177979523$ -ve

[x_0 =

S.NO	a(+ve)	b(-ve)	$x_n = \frac{a+b}{2}$
1	0	1	0.5 (+ve)
2	0.5	1	0.75 (-ve)]

$x=2$. $f(2) = \cos(2) - 2e^2$

$= -0.416146 - 2(7.3905)$

$= -0.416146 - 14.77811$ -ve

$= -15.194256$

$x=3$, $f(3) = \cos(3) - 3e^3$

$= -0.989992496 - 3(20.08553)$

$= -0.989992496 - 60.2566$

$= -61.24659$ -ve

3. Find the root of the equation $x^3-5x+1=0$ by using bisection method

Solu/ Given

$$f(x) = x^3-5x+1=0$$

$x=0$, $f(0)=0-5(0)+1=1$ +ve

$x=1$, $f(1)=1-5+1=-3$ -ve

$x=2$, $f(2)=8-5(2)+1=-1$ -ve

$x=3$, $f(3)=27-15+1=13$ +ve

$$x_0 = \frac{2+3}{2} = \frac{5}{2} = 2.5$$

The root lies between 2 and 3

S.NO	a(-ve)	b(+ve)	$x_n = \frac{a+b}{2}$
1.	2	3	2.5 (+ve)
2	2	2.5	2.25 (+ve)
3.	2	2.25	2.125 (-ve)
4.	2.125	2.25	2.1875 (+ve)
5.	2.125	2.1875	2.15625 (+ve) 2.1563
6.	2.125	2.1563	2.1407 (+ve)
7.	2.125	2.1407	2.1329 (+ve)
8.	2.125	2.1329	2.129 (+ve)
9	2.125	2.129	2.127 (-ve)
10.	2.127	2.129	2.128 (-ve)
11.	2.128	2.129	2.1285 (+ve)
12.	2.128	2.1285	2.1283 (-ve)
13.	2.1283	2.1285	2.1284 (-ve)
14.	2.1284	2.1285	2.1285 (+ve)
15.	2.1284	2.1285	2.1285

$x_{13} = x_{14} = 2.1285$

$x_{14} = x_{15} = 2.1285$

4. Find the real root of the equation $x\log_{10}^{x} = 1.2$ by using bisection method

Solu] Given

$$f(x) = x\log_{10}^{x} - 1.2$$

$x=0,\ f(0) = 0\log_{10}^{0} - 1.2 = -1.2$ (-ve)

$x=1,\ f(1) = 1\log_{10}^{1} - 1.2$

$= 0 - 1.2 = -1.2$ (-ve)

$x=2,\ f(2) = 2\log_{10}^{2} - 1.2$

$= 2(0.3010) - 1.2$

$= 0.602 - 1.2$

$= -0.598$ (-ve)

$x=3,\ f(3) = 3\log_{10}^{3} - 1.2$ (+ve)

$= 3(0.477) - 1.2$

$= 1.43130 - 1.2$

$= 0.23136$

$$x_0 = \frac{2+3}{2} = 2.5$$

S.NO	a(-ve)	b(+ve)	$x_n = \frac{a+b}{2}$
1	2	3	2.5 (-ve)
2.	2.5	3	2.75 (+ve)
3.	2.5	2.75	2.625 (-ve)
4.	2.625	2.75	2.6875 (-ve)
5.	2.6875	2.75	2.7188 (-ve)
6.	2.7188	2.75	2.7344 (-ve)
7.	2.7344	2.75	2.7422 (+ve)
8.	2.7344	2.7422	2.7383 (-ve)

9.	2.7383	2.7422	2.7403 (-ve)
10.	2.7403	2.7422	2.7413 (+ve)
11.	2.7403	2.7413	2.7408 (+ve)
[12	2.7408	2.7413	2.7411 (+ve)
13	2.7408	2.7411	2.741 (+ve)
14.	2.7408	2.741	2.7409 (+ve)
15.	2.7408	2.7409	2.7409 (+ve)
16.	2.7408	2.7409	2.7409]
		2.7408	2.7406 (-ve)
12.	2.7403	2.7408	2.7407 (+ve)
13.	2.7406	2.7408	2.7407 (+ve)
14.	2.7406	2.7407	

$x_{13} = x_{14} = 2.7407$

5. Find the approximate root of the equation $x - \cos x = 0$ by using bi-section method

Solu) Given

$f(x) = x - \cos x = 0$

$x=0, f(0) = 0 - \cos 0 = -1$ -ve

$x=1, f(1) = 1 - \cos(1) = 1 - 0.5403$ +ve

$= 0.4597$

$x_0 = \frac{0+1}{2} = 0.5$

$x_n = \frac{a+b}{2}$

S.NO	a(-ve)	b(+ve)	$x_n = \frac{a+b}{2}$
			0.5 (-ve)
1.	0	1	0.75 (+ve)
2.	0.5	1	0.625 (-ve)
3.	0.5	0.75	0.6875 (-ve)
4.	0.625	0.75	0.7188 (-ve)
5.	0.6875	0.75	

6	0.7188	0.75	0.7344 (-ve)
7.	0.7344	0.75	0.7422 (+ve)
8.	0.7344	0.7422	0.7383 (-ve)
9.	0.7383	0.7422	0.7403 (+ve)
10.	0.7383	0.7403	0.7393 (+ve)
11.	0.7383	0.7393	0.7388 (-ve)
12.	0.7388	0.7393	0.7391 (+ve)
13.	0.7388	0.7391	0.739 (-ve)
14.	0.739	0.7391	0.7391 (+ve)
15.	0.739	0.7391	0.7391 (+ve)

Date 8/9/18

$$x_{14} = x_{15} = 0.7391$$

Iterative method

1. Find the approximate root of the equation $x^3 - x - 1 = 0$ by using iterative method.

Soln Given

$$f(x) = x^3 - x - 1$$

$x = 0$, $f(0) = 0 - 0 - 1 = -1$ -ve

$x = 1$, $f(1) = 1 - 1 - 1 = -1$ -ve

$x = 2$, $f(2) = 8 - 2 - 1 = 5$ +ve

$\therefore$ The root lies between 1 and 2

$$x_0 = \frac{1+2}{2} = 1.5$$

$$x^3 - x - 1 = 0 \Rightarrow x^3 = 1 + x$$

$$x = \sqrt[3]{1+x} = \phi(x)$$

By iterative method.

$$x_1 = \sqrt[3]{1+x_0}, \quad x_0 = 1.5$$

$$x_1 = \sqrt[3]{1+1.5}$$

$x_1 = \sqrt[3]{2.5}$

$x_1 = 1.3572$

$x_2 = \sqrt[3]{1+x_1}$

$= \sqrt[3]{1+1.3572}$

$x_2 = 1.3309$

$x_3 = \sqrt[3]{1+x_2}$

$= \sqrt[3]{1+1.3309}$

$= \sqrt[3]{2.3309}$

$x_3 = 1.3259$

$x_4 = \sqrt[3]{1+x_3}$

$= \sqrt[3]{1+1.3259}$

$= \sqrt[3]{2.3259}$

$x_4 = 1.3249$

$x_5 = \sqrt[3]{1+x_4}$

$= \sqrt[3]{1+1.3249}$

$= \sqrt[3]{2.3249}$

$x_5 = 1.3248$

$x_6 = \sqrt[3]{1+x_5}$

$= \sqrt[3]{1+1.3248}$

$= \sqrt[3]{2.3248}$

$x_6 = 1.3247$

$\cancel{x_5} = \cancel{x_7} = 1.3247$

$x_7 = \sqrt[3]{1+x_6}$

$= \sqrt[3]{1+1.3247}$

$= \sqrt[3]{2.3247}$

$x_7 = 1.3247$

$x_6 = x_7 = 1.3247$

2. find the approximate root of the equation $x^3 - 5x + 1 = 0$

solu/ $f(x) = x^3 - 5x + 1$

$x=0,\ f(0) = 0 - 5(0) + 1 = 1 + ve$

$x=1,\ f(1) = 1 - 5 + 1 = -3 \quad -ve$

$x=2,\ f(2) = 8 - 10 + 1 = -1 \quad -ve$

$x=3,\ f(3) = 27 - 15 + 1 = 13 \quad +ve$ $\quad \frac{28}{\frac{15}{13}}$

The root lies between 2 and 3

$x_0 = \frac{2+3}{2} = \frac{5}{2} = 2.5$

$x^3 - 5x + 1 = 0 \Rightarrow x^3 = 5x - 1$

$x = \sqrt[3]{5x-1} = \phi(x)$

By iterative method

$x_1 = \sqrt[3]{5x_0 - 1}$

$x_1 = \sqrt[3]{5(2.5) - 1}$

$x_1 = \cancel{3.274}\ \sqrt[3]{12.5 - 1}$

$x_1 = \sqrt[3]{11.5}$

$x_1 = 2.2572$

$x_2 = \sqrt[3]{5x_1 - 1}$

$= \sqrt[3]{5(2.2572) - 1}$

$= \sqrt[3]{11.286 - 1}$

$= \sqrt[3]{10.286}$

$x_2 = 2.1748$

$$x_3 = \sqrt[3]{5x_2 - 1}$$
$$= \sqrt[3]{5(2.1748) - 1}$$
$$= \sqrt[3]{10.874 - 1}$$
$$= \sqrt[3]{9.874}$$
$$x_3 = 2.1453$$
$$x_4 = \sqrt[3]{5x_3 - 1}$$
$$= \sqrt[3]{5(2.1453) - 1}$$
$$= \sqrt[3]{10.7265 - 1}$$
$$= \sqrt[3]{9.7265}$$
$$x_4 = 2.1346$$
$$x_5 = \sqrt[3]{5x_4 - 1}$$
$$= \sqrt[3]{5(2.1346) - 1}$$
$$= \sqrt[3]{10.673 - 1}$$
$$= \sqrt[3]{9.673}$$
$$x_5 = 2.1307$$
$$x_6 = \sqrt[3]{5x_5 - 1}$$
$$= \sqrt[3]{5(2.1307) - 1}$$
$$= \sqrt[3]{10.6535 - 1}$$
$$= \sqrt[3]{9.6535}$$
$$x_6 = 2.1293$$
$$x_7 = \sqrt[3]{5x_6 - 1}$$
$$= \sqrt[3]{5(2.1293) - 1}$$
$$= \sqrt[3]{9.6465}$$
$$= 2.1287$$

$$x_8 = \sqrt[3]{5x_7 - 1}$$
$$= \sqrt[3]{5(2.1287) - 1}$$
$$= \sqrt[3]{9.6435}$$
$$x_8 = 2.1285$$
$$x_9 = \sqrt[3]{5x_8 - 1}$$
$$= \sqrt[3]{5(2.1285) - 1}$$
$$= \sqrt[3]{9.6425}$$
$$x_9 = 2.1284$$
$$x_{10} = \sqrt[3]{5x_9 - 1}$$
$$= \sqrt[3]{5(2.1284) - 1}$$
$$= \sqrt[3]{9.642}$$
$$x_{10} = 2.1284$$
$$x_9 = x_{10} = 2.1284$$

3. find the approximate root of the equation $\cos x = 3x - 1$

Solu) $f(x) = \cos x - 3x + 1$

$x = 0$, $f(0) = \cos 0 - 3(0) + 1$ +ve
$= 1 + 1 = 2$

$x = 1$, $f(1) = \cos(1) - 3 + 1$ -ve
$= 0.54030 - 3 + 1$
$= -1.459697$

The root lies between 0 and 1

$$x_0 = \frac{1+0}{2} = \frac{1}{2} = 0.5$$

$\cos x - 3x + 1 = 0 \Rightarrow \cos x + 1 = 3x$

$$x = \frac{1 + \cos x}{3} = \phi(x)$$

By iterative method

$$x_1 = \frac{1+\cos x_0}{3}$$

$$x_1 = \frac{1+\cos(0.5)}{3}$$

$$x_1 = 0.6259$$

$$x_2 = \frac{1+\cos x_1}{3}$$

$$= \frac{1+\cos(0.6259)}{3}$$

$$= \frac{1+0.810436203}{3}$$

$$x_2 = 0.6035$$

$$x_3 = \frac{1+\cos x_2}{3}$$

$$= \frac{1+\cos(0.6035)}{3}$$

$$= \frac{1+0.823354315}{3}$$

$$x_3 = 0.6078$$

$$x_4 = \frac{1+\cos x_3}{3}$$

$$= \frac{1+\cos(0.6078)}{3}$$

$$= \frac{1+0.820906341}{3}$$

$$x_4 = 0.607$$

$$x_5 = \frac{1+\cos x_4}{3}$$

$$= \frac{1+\cos(0.607)}{3}$$

$$= \frac{1+0.821362929}{3}$$

$$x_5 = 0.6071$$

$$x_6 = \frac{1+\cos x_5}{3}$$

$$= \frac{1+\cos(0.6071)}{3}$$

$$x_6 = 0.6071$$

$$x_5 = x_6 = 0.6071$$

4. Find the approximate value of $x^3+x^2 \pm 1 = 0$

$$f(x) = x^3+x^2-1$$

$x=0$, $f(0) = 0+0-1 = -1$ $-ve$

$x=1$, $f(1) = 1+1-1 = +1$ $+ve$

The root lies between 0 and 1

$$x_0 = \frac{0+1}{2} = \frac{1}{2} = 0.5$$

$[x^3+x^2 \pm 1 = 0$

$x^2(x+1 \pm 1) = 0$

$x^2(\cdot \; x^3+x^2]$

$$x^3+x^2-1=0$$

$$x^3+x^2=1$$

$$x^2(x+1)=1$$

$$x^2 = \frac{1}{1+x}$$

By iteration method $x = \frac{1}{\sqrt{1+x}} = \phi(x)$

$$x_1 = \frac{1}{\sqrt{1+x_0}} = \frac{1}{\sqrt{1+0.5}}$$

$$= \frac{1}{\sqrt{1.5}} = \frac{1}{1.224744871} = 0.8165$$

$$x_2 = \frac{1}{\sqrt{1+x_1}} = \frac{1}{\sqrt{1+0.8165}} = \frac{1}{\sqrt{1.8165}} = 0.742$$

$$x_3 = \frac{1}{\sqrt{1+x_2}} = \frac{1}{\sqrt{1+0.742}} = \frac{1}{\sqrt{1.742}} = 0.7577$$

$$x_4 = \frac{1}{\sqrt{1+x_3}} = \frac{1}{\sqrt{1+0.7577}} = \frac{1}{\sqrt{1.7577}} = 0.7543$$

$$x_5 = \frac{1}{\sqrt{1+x_4}} = \frac{1}{\sqrt{1+0.7543}} = \frac{1}{\sqrt{1.7543}} = 0.7550$$

$$x_6 = \frac{1}{\sqrt{1+x_5}} = \frac{1}{\sqrt{1+0.7550}} = \frac{1}{\sqrt{1.7550}} = 0.755$$

Date 11/9/18

5. Find a root near 3.8 for the equation $2x - \log_{10}^{x} = 7$ correct to 4 decimal places by the iterative method

Solu) $f(x) = 2x - \log_{10}^{x} - 7$

Given $x_0 = 3.8$

$$2x - \log_{10}^{x} = 7$$

$$2x = \log_{10}^{x} + 7$$

$$x = \frac{1}{2}[\log_{10}^{x} + 7] = \phi(x)$$

By iterative method

$$x_1 = \frac{1}{2}[\log_{10}^{x_0} + 7]$$

$$x_1 = \frac{1}{2}[\log_{10}^{(3.8)} + 7]$$

$$= 3.789891798$$

$$x_1 = 3.7899$$

$$x_2 = \frac{1}{2}[\log_{10}^{x_1} + 7]$$

$$= \frac{1}{2}[\log_{10}^{(3.7899)} + 7]$$

$$= 3.789313875$$

$$x_2 = 3.7893$$

$$x_3 = \frac{1}{2}[\log_{10}^{x_2} + 7]$$

$$= \frac{1}{2}[\log_{10}^{(3.7893)} + 7]$$

$$= 3.789279495$$

$$x_3 = 3.7893$$

$x_2 = x_3 = 3.7893$

6. Find the approximate root of the equation $\tan x = x$ by using iterative method

$f(x) = \tan x - x$

$x=0,\ f(0) = \tan 0 - 0 = 0$ +ve

$x=1,\ f(1) = \tan 1 - 1 = 0.557407724$ +ve

$x=2,\ f(2) = \tan 2 - 2 = -4.185039$ −ve

The root lies between 1 ∧ 2

$x_0 = \frac{1+2}{2} = \frac{3}{2} = 1.5$

$\tan x = x = \phi(x)$

$[x_1 = \tan x_0 = \tan(1.5)$

$= 14.10141995$

$= 14.1014$

$x_2 = \tan x_1]$

$x = \tan^{-1}(x)$

$x_1 = \tan^{-1} x_0$

$= \tan^{-1}(1.5)$

$= 0.982793723$

$x_1 = 0.9828$

$x_2 = \tan^{-1}(x_1)$

$= \tan^{-1}(0.9828)$

$= 0.776723779)$

$x_2 = 0.7767$

$x_3 = \tan^{-1}(x_2)$

$= \tan^{-1}(0.7767)$

$= 0.660371299$

$x_3 = 0.6604$

$x_4 = \tan^{-1}(x_3)$

$= \tan^{-1}(0.6604)$

$= 0.583651584$

$x_4 = 0.5837$

$x_5 = \tan^{-1}(x_4)$

$= \tan^{-1}(0.5837)$

$= 0.528347979$

$x_5 = 0.5283$

$x_6 = \tan^{-1}(x_5)$

$= \tan^{-1}(0.5283)$

$= 0.486030454$

$x_6 = 0.4860$

$x_7 = \tan^{-1}(x_6)$

$= \tan^{-1}(0.4860)$

$= 0.452385012$

$x_7 = 0.4524$

$x_8 = \tan^{-1}(x_7)$

$= \tan^{-1}(0.4524)$

$= 0.424847974$

$x_8 = 0.4248$

$x_9 = \text{ban}^{-1}(x_8)$

$= \tan^{-1}(0.4248)$

$= 0.401701233$

$x_9 = 0.4017$

$x_{10} = \tan^{-1}(x_9)$

$= \tan^{-1}(0.4017)$

$= 0.381971034$

$x_{10} = 0.382$

$x_{11} = \tan^{-1}(x_{10})$

$= \tan^{-1}(0.382)$

$= 0.36$[illegible]489

$= 0.3649$

$x_{12} = \tan^{-1}(x_{11})$

$= \tan^{-1}(0.3649)$

$= 0.349886608$

$x_{12} = 0.3499$

$x_{13} = \tan^{-1}(x_{12})$

$= \tan^{-1}(0.3499)$

$= 0.336585729$

$x_{13} = 0.3366$

$x_{14} = \tan^{-1}(0.3366) = \tan^{-1}(x_{13})$

$= 0.324687667$

$x_{14} = 0.3247$

$x_{15} = \tan^{-1}(x_{14})$

$= \tan^{-1}(0.3247)$

$= 0.313960535$

$x_{15} = 0.3140$

$x_{16} = \tan^{-1}(x_{15})$

$= \tan^{-1}(0.3140)$

$= 0.304250832$

$x_{16} = 0.3043$

$x_{17} = \tan^{-1}(x_{16})$

$= \tan^{-1}(0.3043)$

$= 0.295397064$

$x_{17} = 0.2954$

$x_{18} = \tan^{-1}(x_{17})$

$= \tan^{-1}(0.2954)$

$= 0.287231286$

$x_{19} = \tan^{-1}(x_{18})$

$= \tan^{-1}(0.2872)$

$= 0.279672704$

$x_{19} = 0.2797$

$x_{20} = \tan^{-1}(x_{19})$

$= \tan^{-1}(0.2797)$

$= 0.272730491$

$x_{20} = 0.2727$

$x_{21} = \tan^{-1}(x_{20})$

$= \tan^{-1}(0.2727)$

$= 0.266226664$

$x_{21} = 0.2662$

$x_{22} = \tan^{-1}(x_{21})$

$= \tan^{-1}(0.2662)$

$= 0.260166056$

$x_{22} = 0.2602$

$x_{23} = \tan^{-1}(x_{22})$

$= \tan^{-1}(0.2602)$

$= 0.254555385$

$x_{23} = 0.2546$

$x_{24} = \tan^{-1}(x_{23})$

$= \tan^{-1}(0.2546)$

$= 0.249303367$

$x_{24} = 0.2493$

$x_{25} = \tan^{-1}(x_{24})$

$= \tan^{-1}(0.2493)$

= 0.244[illegible]731

$x_{25} = 0.2443$

$x_{26} = \tan^{-1}(x_{25})$

$= \tan^{-1}(0.2443)$

$= 0.239606804$

$x_{26} = 0.2396$

$x_{27} = \tan^{-1}(x_{26})$

$= \tan^{-1}(0.2396)$

$x_{27} = 0.2352$

$x_{28} = \tan^{-1}(x_{27})$

$= \tan^{-1}(0.2352)$

$x_{28} = 0.231$

$x_{29} = \tan^{-1}(x_{28})$

$= \tan^{-1}(0.231)$

$x_{29} = 0.2270$

$x_{30} = \tan^{-1}(x_{29})$

$= \tan^{-1}(0.2270)$

$x_{30} = 0.2232$

$x_{31} = \tan^{-1}(x_{30})$

$= \tan^{-1}(0.2232)$

$x_{31} = 0.2196$

$x_{32} = \tan^{-1}(x_{31})$

$= \tan^{-1}(0.2196)$

$x_{32} = 0.2162$

$x_{33} = \tan^{-1}(x_{32})$

$= \tan^{-1}(0.2162)$

$x_{42} = \tan^{-1}(x_{41})$

$= \tan^{-1}(0.1911)$

$x_{42} = 0.1888$

$x_{43} = \tan^{-1}(x_{42})$

$= \tan^{-1}(0.1888)$

$x_{43} = 0.1866$

$x_{44} = \tan^{-1}(x_{43})$

$= \tan^{-1}(0.1866)$

$x_{44} = 0.1845$

$x_{45} = \tan^{-1}(x_{44})$

$= \tan^{-1}(0.1845)$

$x_{45} = 0.1824$

$x_{46} = \tan^{-1}(x_{45})$

$= \tan^{-1}(0.1824)$

$x_{46} = 0.1804$

$x_{47} = \tan^{-1}(x_{46})$

$= \tan^{-1}(0.1804)$

$x_{47} = 0.1785$

$x_{48} = \tan^{-1}(x_{47})$

$= \tan^{-1}(0.1785)$

$x_{48} = 0.1766$

$x_{49} = \tan^{-1}(x_{48})$

$= \tan^{-1}(0.1766)$

$x_{49} = 0.1748$

$= 0.2129$

$x_{34} = \tan^{-1}(x_{33})$

$= \tan^{-1}(0.2129)$

$x_{34} = 0.2098$

$x_{35} = \tan^{-1}(x_{34})$

$= \tan^{-1}(0.2098)$

$x_{35} = 0.2068$

$x_{36} = \tan^{-1}(x_{35})$

$x = \tan^{-1}(0.2068)$

$x_{36} = 0.2037$

$x_{37} = \tan^{-1}(x_{36})$

$= \tan^{-1}(0.2037)$

$x_{37} = 0.2011$

$x_{38} = \tan^{-1}(x_{37})$

$= \tan^{-1}(0.2011)$

$x_{38} = 0.1985$

$x_{39} = \tan^{-1}(x_{38})$

$= \tan^{-1}(0.1985)$

$x_{39} = 0.1960$

$x_{40} = \tan^{-1}(39)$

$= \tan^{-1}(0.1960)$

$x_{40} = 0.1935$

$x_{41} = \tan^{-1}(x_{40})$

$= \tan^{-1}(0.1935)$

$x_{41} = 0.1911$

$x_{50} = \tan^{-1}(x_{49})$

$= \tan^{-1}(0.1748)$

$x_{50} = 0.1731$

$x_{51} = \tan^{-1}(x_{50})$

$= \tan^{-1}(0.1731)$

$x_{51} = 0.1714$

$x_{52} = \tan^{-1}(x_{51})$

$= \tan^{-1}(0.1714)$

$x_{52} = 0.1698$

$x_{53} = \tan^{-1}(x_{52})$

$= \tan^{-1}(0.1698)$

$x_{53} = 0.1682$

$x_{54} = \tan^{-1}(x_{53})$

$= \tan^{-1}(0.1682)$

$x_{54} = 0.1666$

$x_{55} = \tan^{-1}(x_{54})$

$= \tan^{-1}(0.1666)$

$x_{55} = 0.1651$

$x_{56} = \tan^{-1}(x_{55})$

$= \tan^{-1}(0.1651)$

$x_{56} = 0.1636$

$x_{57} = \tan^{-1}(x_{56})$

$= \tan^{-1}(0.1636)$

$x_{57} = 0.1622$

$x_{58} = \tan^{-1}(x_{57})$

$= \tan^{-1}(0.1622)$

$x_{58} = 0.1608$

$x_{59} = \tan^{-1}(x_{58})$

$= \tan^{-1}(0.1608)$

$x_{59} = 0.1594$

$x_{60} = \tan^{-1}(x_{59})$

$= \tan^{-1}(0.1594)$

$x_{60} = 0.1587$

$x_{61} = \tan^{-1}(x_{61})$

$= \tan^{-1}(0.1587)$

$x_{61} = 0.1568$

$x_{62} = \tan^{-1}(x_{61})$

$= \tan^{-1}(0.1568)$

$x_{62} = 0.1555$

$x_{63} = \tan^{-1}(x_{62})$

$= \tan^{-1}(0.1555)$

$x_{63} = 0.1543$

$x_{64} = \tan^{-1}(x_{63})$

$= \tan^{-1}(0.1543)$

$x_{64} = 0.1531$

$x_{65} = \tan^{-1}(x_{64})$

$= \tan^{-1}(0.1531)$

$x_{65} = 0.1519$

$x_{66} = \tan^{-1}(x_{65})$

$= \tan^{-1}(0.1519)$

$x_{66} = 0.1507$

$x_{67} = \tan^{-1}(x_{66})$

$= \tan^{-1}(1507)$

$x_{67} = 0.1496$

$x_{68} = \tan^{-1}(x_{67})$

$= \tan^{-1}(0.1496)$

$x_{68} = 0.1485$

$x_{69} = \tan^{-1}(x_{68})$

$= \tan^{-1}(0.1485)$

$x_{69} = 0.1474$

$x_{70} = \tan^{-1}(x_{69})$

$= \tan^{-1}(0.1474)$

$x_{70} = 0.1463$

$x_{71} = \tan^{-1}(x_{70})$

$= \tan^{-1}(0.1463)$

$x_{71} = 0.1453$

$x_{72} = \tan^{-1}(x_{71})$

$= \tan^{-1}(0.1453)$

$x_{72} = 0.1443$

$x_{73} = \tan^{-1}(x_{72})$

$= \tan^{-1}(0.1443)$

$x_{74} = 0.1433$

$x_{74} = \tan^{-1}(x_{74})$

$= \tan^{-1}(0.1433)$

$x_{74} = 0.1423$

$x_{75} = \tan^{-1}(x_{74})$

$= \tan^{-1}(0.1423)$

$x_{75} = 0.1414$

$x_{76} = \tan^{-1}(x_{76})$
$= \tan^{-1}(0.1414)$

$x_{76} = 0.1405$

$x_{77} = \tan^{-1}(x_{76})$
$= \tan^{-1}(0.1405)$

$x_{77} = 0.1396$

$x_{78} = \tan^{-1}(x_{77})$
$= \tan^{-1}(0.1396)$

$x_{78} = 0.1387$

$x_{79} = \tan^{-1}(x_{78})$
$= \tan^{-1}(0.1387)$

$x_{79} = 0.1378$

$x_{80} = \tan^{-1}(x_{79})$
$= \tan^{-1}(0.1378)$

$x_{80} = 0.1369$

$x_{81} = \tan^{-1}(x_{80})$
$= \tan^{-1}(0.1369)$

$x_{81} = 0.1361$

$x_{82} = \tan^{-1}(x_{81})$
$= \tan^{-1}(0.1361)$

$x_{82} = 0.1353$

$x_{83} = \tan^{-1}(x_{82})$
$= \tan^{-1}(0.1353)$

$x_{83} = 0.1345$

$x_{84} = \tan^{-1}(x_{83})$
$= \tan^{-1}(0.1345)$
$= 0.1337$

$x_{85} = \tan^{-1}(x_{84})$
$= \tan^{-1}(0.1337)$

$x_{85} = 0.1329$

$x_{86} = \tan^{-1}(x_{85})$
$= \tan^{-1}(0.1329)$

$x_{86} = 0.1321$

$x_{87} = \tan^{-1}(x_{86})$
$= \tan^{-1}(0.1321)$

$x_{87} = 0.1313$

$x_{88} = \tan^{-1}(x_{87})$
$= \tan^{-1}(0.1313)$

$x_{88} = 0.1306$

$x_{89} = \tan^{-1}(x_{88})$
$= \tan^{-1}(0.1306)$

$x_{89} = 0.1299$

$x_{90} = \tan^{-1}(x_{89})$
$= \tan^{-1}(0.1299)$

$x_{90} = 0.1292$

$x_{91} = \tan^{-1}(x_{90})$
$= \tan^{-1}(0.1292)$

$x_{91} = 0.1285$

$x_{92} = \tan^{-1}(x_{91})$
$= \tan^{-1}(0.1285)$

$x_{92} = 0.1278$

$x_{93} = \tan^{-1}(92)$
$= \tan^{-1}(0.1278)$

$x_{93} = 0.1271$

$x_{94} = \tan^{-1}(x_{93})$

$= \tan^{-1}(0.1271)$

$x_{94} = 0.1264$

$x_{95} = \tan^{-1}(x_{94})$

$= \tan^{-1}(0.1264)$

$x_{95} = 0.1257$

$x_{96} = \tan^{-1}(x_{95})$

$= \tan^{-1}(0.1257)$

$x_{96} = 0.1250$

$x_{97} = \tan^{-1}(x_{96})$

$= \tan^{-1}(0.1250)$

$x_{97} = 0.1244$

$x_{98} = \tan^{-1}(x_{97})$

$= \tan^{-1}(0.1244)$

$x_{98} = 0.1238$

$x_{99} = \tan^{-1}(98)$

$= \tan^{-1}(0.1238)$

$x_{99} = 0.1232$

$x_{100} = \tan^{-1}(x_{99})$

$= \tan^{-1}(0.1232)$

$x_{100} = 0.1226$

Date 13/8/18

1. Solutions of Algebraic & Transcendental Equations.

Since the given equation having trignometric functions or logarithmic functions or exponent functions, that type of equations are called 'transcendental' equations

Ex:

1. $x = e^{-x}$
2. $x+1 = \log x$
3. $x = \sin x + 1$

In the given linear equation having 'x' is called algebraic equation.

Ex:

1. $x^2+x+1 = 0$
2. $x^3 - 2x^2+x+1 = 0$

⇒ <u>Newton - Rathson</u> <u>Method</u> (or) <u>Newton's</u> <u>Method</u>.

Consider $f(x) = 0$ be the given curve and x takes the values $x_0, x_1, x_2 --- x_n$, and h is the common difference then $x_1 = x_0+h \rightarrow ①$

By taylor's Series

$$f(x+h) = f(x)+h.f'(x) + \frac{h^2}{2!} f''(x) + \frac{h^3}{3!} f'''(x) + \cdots$$

Since 'h' is very small quantity and $h^2, h^3, h^4, \ldots$ are very small [negligible]

∴ In the above Equation we eliminate the product of $h^2, h^3, h^4, \ldots$ terms. then $f(x+h) = f(x)+hf'(x)$

If $x = x_1$ is the solution of the given Equation $f(x_1)=0$

$$\Rightarrow f(x_0+h) = 0$$

$$\Rightarrow f(x_0+h) = f(x_0) + hf'(x_0)$$

$$\Rightarrow h \cdot f'(x_0) = -f(x_0)$$

then $h = -\dfrac{f(x_0)}{f'(x_0)} \rightarrow ②$

From ① & ②

$$x_1 = x_0 + \left[\frac{-f(x_0)}{f'(x_0)} \right]$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$ similarly:

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_0)} \quad ; \quad x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$\boxed{\therefore x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}$$

The above equation is called "Newton's Formulae".

Geometrical Representation of Newton's Formulae

Consider the curve $y = f(x)$ be passing through the points (x_0, y_0), (x_1, y_1).

The slope of the curve $m = \frac{dy}{dx} = f'(x)$

It passing

At (x_0, y_0) $m = f'(x_0) \rightarrow$ ①

The given line (or) curve passing through (x_0, y_0) and slope $m = f'(x_0)$ then Equation to the line

$$y - y_0 = m(x - x_0)$$

$$\Rightarrow y - y_0 = f'(x_0)(x - x_0)$$

it intersect x-axis then it's y-co-ordinate is zero.

$$\therefore 0 - y_0 = f'(x_0)(x_1 - x_0)$$

$$-y_0 = f'(x_0)(x_1 - x_0)$$

$$x_1 - x_0 = \frac{-y_0}{f'(x_0)}$$

$$x_1 = x_0 - \frac{y_0}{f'(x_0)}, \quad y_0 = f(x_0)$$

$$\therefore x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

similarly $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$; $x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$

$$\therefore x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

1. Using Newtons Rafhson method, find the real root of the situation $3x = \cos x + 1$ correct to four decimal places

Solu

$$3x = \cos x + 1$$

$$f(x) = 3x - \cos x - 1$$

$x=0 \Rightarrow f(0) = 3(0) - \cos 0 - 1$

$= 0 - 1 - 1 = -2$ -ve

$x=1 \Rightarrow f(1) = 3(1) - \cos 1 - 1$

$= 3 - 0.9998 - 1$ +ve

$$x_0 = \frac{a+b}{2} = \frac{0+1}{2} = 0.5$$

$$f(x) = 3x - \cos x - 1$$

$$f'(x) = \frac{d}{dx}(3x - \cos x - 1)$$

$$= 3 - (-\sin x)$$

$$= 3 + \sin x$$

By Newton's Method

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= x_0 - \frac{(3x_0 - \cos x_0 - 1)}{3 + \sin x_0}$$

$$= \frac{x_0(3 + \sin x_0) - (3x_0 - \cos x_0 - 1)}{3 + \sin x_0}$$

$$= \frac{\cancel{3x_0} + x_0 \sin x_0 - \cancel{3x_0} + \cos x_0 + 1}{3 + \sin x_0}$$

$$x_1 = \frac{x_0 \sin x_0 + \cos x_0 + 1}{3 + \sin x_0}$$

$$x_1 = \frac{0.5 \sin(0.5) + \cos(0.5) + 1}{3 + \sin(0.5)}$$

$$= \frac{2.11729533}{3.4794255386}$$

$$= 0.6085186498$$

$$x_1 = \cancel{0.6662}\ 0.6085$$

$$x_2 = \frac{x_1 \sin x_1 + \cos x_1 + 1}{3 + \sin x_1}$$

$$= \frac{(0.6085) \sin(0.6085) + \cos(0.6085) + 1}{3 + \sin(0.6085)}$$

$$= \frac{2.1956929118}{3.6146956091} \quad \left[= \frac{0.6085[0.010620128] + 0.9999436044 + 1}{3 + 0.010620128}\right.$$

$$= \frac{0.0064623484 07 + 0.99994360 4 + 1}{3 + 0.010620128}$$

$$\left.= \frac{2.006405952}{3.010620128}\right]$$

$$x_2 = 0.6071087$$

$$x_3 = \frac{x_2 \sin x_2 + \cos x_2 + 1}{3 + \sin x_2}$$

$$= \frac{(0.6071) \sin(0.6071) + \cos(0.6071) + 1}{3 + \sin(0.6071)}$$

$$= \frac{0.6071 \left[\cancel{0.010595 69562}\ 0.570488075\right] + 0.8213058884 + 1}{3 + 0.570488075}$$

$$= \frac{0.34634331 + 1 + 0.821305884}{3.570488075} = \frac{2.167649194}{3.570488075}$$

$$= 0.607101647$$

$x_2 = x_3 = 0.6071$

The approximate root of the given equation is 0.6071

2. Find the real root of the equation $x = e^{-x}$ by using Newton Raphson method

Solu)

$x_1 = x_0 - f$

$x = 0 \Rightarrow f(0) = 0 - e^{-0} = -1$ $-ve$

$x = 1 \Rightarrow f(1) = 1 - e^{-1} = 0.6321$ $+ve$

$$x_0 = \frac{a+b}{2} = \frac{0+1}{2} = 0.5$$

$f(x) = x - e^{-x}$

$$f'(x) = \frac{d}{dx}[x - e^{-x}]$$

$$= 1 - e^{-x}(-1)$$

$$= 1 + e^{-x}$$

By Newton's method

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= x_0 - \frac{(x_0 - e^{-x_0})}{1 + e^{-x_0}}$$

$$= \frac{x_0(1 + e^{-x_0}) - (x_0 - e^{-x_0})}{1 + e^{-x_0}}$$

$$= \frac{\cancel{x_0} + x_0 e^{-x_0} \cancel{- x_0} + e^{-x_0}}{1 + e^{-x_0}}$$

$$x_1 = \frac{e^{-x_0}(x_0 + 1)}{1 + e^{-x_0}} \qquad x_0 = 0.5$$

$$x_1 = \frac{e^{-0.5}(0.5+1)}{1 + e^{-0.5}}$$

$$= \frac{0.606530659(1.5)}{1 + 0.606530659}$$

$$= \frac{0.909795988}{1.606530659}$$

$$= 0.5663110031$$

$$x_1 = 0.5663$$

$$x_2 = \frac{e^{-x_1}(x_1+1)}{1+e^{-x_1}}$$

$$= \frac{e^{-0.5663}(0.5663+1)}{1+e^{-0.5663}}$$

$$= \frac{0.5676217586(1.5663)}{1+0.5676217586}$$

$$= \frac{0.8890659605}{1.567621759}$$

$$= 0.5671$$

$$x_3 = \frac{e^{-x_2}(x_2+1)}{1+e^{-x_2}}$$

$$= \frac{e^{-0.5671}(0.5671+1)}{1+e^{-0.5671}}$$

$$= \frac{0.5671678428(1.5671)}{1+0.5671678428}$$

$$= \frac{0.8888087265}{1.567167843}$$

$$= 0.56714329$$

$$= 0.5671$$

$$x_2 = x_3 = 0.5671$$

The approximate roots of the given Equation 0.5671

Date 18/8/18

3\. Find the approximate root of the equation $x^3-5x+3=0$ by using Newtons method.

Solu/ Given

$$x^3-5x+3=0$$

$$f(x) = x^3-5x+3$$

$x=0 \Rightarrow 0-5(0)+3 = 3$ +ve

$x=1 \Rightarrow 1-5(1)+3 = -1$ -ve

$x=2 \Rightarrow 2^3-5(2)+3 = 8-10+3 = 1$ +ve

$$x_0 = \frac{a+b}{2} = \frac{1+2}{2} = \frac{3}{2} = 1.5 \quad ; a=1,\ b=2$$

root lies between 1 and 2

$$f(x) = x^3-5x+3$$

$$f'(x) = 3x^2-5$$

By Newton's method

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= x_0 - \frac{[x_0^3-5x_0+3]}{3x_0^2-5}$$

$$= \frac{x_0(3x_0^2-5)-(x_0^3-5x_0+3)}{3x_0^2-5}$$

$$= \frac{3x_0^3-\cancel{5x_0}-x_0^3+\cancel{5x_0}-3}{3x_0^2-5}$$

$$x_1 = \frac{2x_0^3-3}{3x_0^2-5} \qquad x_0 = 1.5$$

$$x_1 = \frac{2(1.5)^3-3}{3(1.5)^2-5} = \frac{2(3.375)-3}{3(2.25)-5} = \frac{6.75-3}{6.75-5}$$

$$= \frac{3.75}{1.75} = 2.142857143$$

$$x_1 = 2.1429$$

$$x_2 = \frac{2x_1^3 - 3}{3x_1^2 - 5} \qquad x_1 = 2.1429$$

$$x_2 = \frac{2(2.1429)^3 - 3}{3(2.1429)^2 - 5} = \frac{2(9.840240537) - 3}{3(4.59202041) - 5}$$

$$= \frac{19.68048107 - 3}{13.77606123 - 5} = \frac{16.68048107}{8.77606123}$$

$$= 1.900679659$$

$$x_2 = 1.9007$$

$$x_3 = \frac{2x_2^3 - 3}{3x_2^2 - 5} \qquad x_2 = 1.9007$$

$$= \frac{2(1.9007)^3 - 3}{3(1.9007)^2 - 5} = \frac{2(6.866583793) - 3}{3(3.61266049) - 5}$$

$$= \frac{13.73316759 - 3}{10.83798147 - 5} = \frac{10.73316759}{5.83798147}$$

$$= 1.838506622$$

$$x_3 = 1.8385$$

$$x_4 = \frac{2x_3^3 - 3}{3x_3^2 - 5}$$

$$= \frac{2(1.8385)^3 - 3}{3(1.8385)^2 - 5} = \frac{2(6.214281217) - 3}{3(3.38008225) - 5}$$

$$= \frac{12.428562433 - 3}{10.14024675 - 5} = \frac{9.428562433}{5.14024675}$$

$$= 1.834262613$$

$$x_4 = 1.8343$$

$$x_5 = \frac{2x_4^3 - 3}{3x_4^2 - 5} = \frac{2(1.8343)^3 - 3}{3(1.8343)^2 - 5}$$

$$= \frac{2(6.1717894) - 3}{3(3.36465649) - 5} = \frac{12.3435788 - 3}{10.09396947 - 5}$$

$= 9.4265$

$$= \frac{9.3435788}{5.0939694 7}$$

$= 1.834243188$

$= 1.8342$

$$x_6 = \frac{2x_5^3 - 3}{3x_5^2 - 5}$$

$$= \frac{2(1.8342)^3 - 3}{3(1.8342)^2 - 5}$$

$$= \frac{2(6.170780058) - 3}{3(3.36428964) - 5}$$

$$= \frac{12.34156012 - 3}{10.09286892 - 5}$$

$$= \frac{9.341560116}{5.09286892}$$

$= 1.834243186$

$= 1.8342$

The approximate roots $x_5 = x_6 = 1.8342$

4. Find the real root of the equation $x^3 - 2x - 5 = 0$ by using Newton's method.

Solu) Given Equation

$x^3 - 2x - 5 = 0$

$f(x) = x^3 - 2x - 5$

$x = 0 \Rightarrow 0 - 2(0) - 5 = -5$ −ve

$x = 1 \Rightarrow 1 - 2(1) - 5 = -6$ −ve

$x = 2 \Rightarrow 2^3 - 2(2) - 5 = 8 - 4 - 5 = -1$ −ve

$x = 3 \Rightarrow 3^3 - 2(3) - 5 = 27 - 6 - 5 = 16$ +ve

$$x_0 = \frac{a+b}{2} = \frac{2+3}{2} = \frac{5}{2} = 2.5$$

$f(x) = x^3 - 2x - 5$

$f'(x) = 3x^2 - 2$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= x_0 - \frac{[x_0^3 - 2x_0 - 5]}{(3x_0^2 - 2)}$$

$$= \frac{x_0(3x_0^2 - 2) - [x_0^3 - 2x_0 - 5]}{3x_0^2 - 2}$$

$$= \frac{3x_0^3 - \cancel{2x_0} - x_0^3 + \cancel{2x_0} + 5}{3x_0^2 - 2}$$

$$x_1 = \frac{2x_0^3 + 5}{3x_0^2 - 2}$$

$$x_1 = \frac{2(2.5)^3 + 5}{3(2.5)^2 - 2}$$

$$= \frac{2(15.625) + 5}{3(6.25) - 2} = \frac{31.25 + 5}{18.75 - 2}$$

$$= \frac{36.25}{16.75} = 2.164179104$$

$$x_1 = 2.1642$$

$$x_2 = \frac{2x_1^3 + 5}{3x_1^2 - 2} = \frac{2(2.1642)^3 + 5}{3(2.1642)^2 - 2}$$

$$= \frac{2(10.13659694) + 5}{3(4.68376164) - 2}$$

$$= \frac{20.27319388 + 5}{14.05128492 - 2}$$

$$= \frac{25.27319388}{12.05128492}$$

$$= 2.097136865$$

$$= 2.0971$$

$$x_3 = \frac{2x_2^3+5}{3x_2^2-2}$$

$$= \frac{2(2.0971)^3+5}{3(2.0971)^2-2}$$

$$= \frac{2(9.222685959)+5}{3(4.39782841)-2}$$

$$= \frac{18.44537192+5}{13.19348523-2}$$

$$= \frac{23.44537192}{11.19348523}$$

$$= 2.094555131$$

$$x_3 = 2.0946$$

$$x_4 = \frac{2x_3^3+5}{3x_3^2-2}$$

$$= \frac{2(2.0946)^3+5}{3(2.0946)^2-2}$$

$$= \frac{2(9.189741551)+5}{3(4.38734916)-2}$$

$$= \frac{18.3794831+5}{13.16204748-2}$$

$$= \frac{23.3794831}{11.16204748}$$

$$= 2.09455148З$$

$$x_4 = 2.0946$$

The approximate roots are $x_3 = x_4 = 2.0946$

H.W 5. find the real root of the equation $x^4-x-10=0$ by which is near to $x=2$.

Solu| Given that

$x^4 x - 10 = 0$ $\quad f(x) = x^4 - x - 10 = 0$

$x_0 = 2$ $\quad f'(x) = 4x^3 - 1$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= x_0 - \frac{(x_0^4 - x_0 - 10)}{(4x_0^3 - 1)}$$

$$= \frac{x_0(4x_0^3 - 1) - (x_0^4 - x_0 - 10)}{4x_0^3 - 1}$$

$$= \frac{4x_0^4 - \cancel{x_0} - x_0^4 + \cancel{x_0} + 10}{4x_0^3 - 1}$$

$$x_1 = \frac{3x_0^4 + 10}{4x_0^3 - 1} \qquad x_0 = 2$$

$$x_1 = \frac{3(2)^4 + 10}{4(2)^3 - 1} = \frac{3(16) + 10}{4(8) - 1} = \frac{48 + 10}{32 - 1} = \frac{58}{31}$$

$$= 1.870967742$$

$$x_1 = 1.879$$

$$x_2 = \frac{3x_1^4 + 10}{4x_1^3 - 1} = \frac{3(1.871)^4 + 10}{4(1.871)^3 - 1}$$

$$= \frac{3(12.25448741) + 10}{4(6.549699311) - 1}$$

$$= \frac{36.76346223 + 10}{26.19879724 - 1}$$

$$= \frac{46.76346223}{25.19879724}$$

$$= 1.85578152$$

$$x_2 = 1.8558$$

$$x_3 = \frac{3x_2^4 + 10}{4x_2^3 - 1} = \frac{3(1.8558)^4 + 10}{4(1.8558)^3 - 1}$$

$$= \frac{3(11.86109219) + 10}{4(6.391363397) - 1} = \frac{35.58327658 + 10}{25.56545359 - 1}$$

$$= \frac{45.58327658}{24.56545359}$$

$$= 1.855584568$$

$$x_3 = 1.8556$$

$$x_4 = \frac{3x_3^4+10}{4x_3^3-1} = \frac{3(1.8556)^4+10}{4(1.8556)^3-1}$$

$$= \frac{3(11.85597993)+10}{4(6.389297224)-1}$$

$$= \frac{35.56793978+10}{25.55718889-1}$$

$$= \frac{45.56793978}{24.55718889}$$

$$= 1.855584529$$

$$x_4 = 1.8556$$

The approximate roots are $x_3 = x_4 = 1.8556$

Date 17/8/18 Logarithm functions

6. find the real root of the equation $x \log_{10}^{x} = 1.2$

Solu) Given

$$f(x) = x \log_{10}^{x} - 1.2$$

Put $x=0$

$x=0$, $f(0) = 0\log_{10}^{0} - 1.2 = -1.2$ −ve

$x=1$, $f(1) = 1\log_{10}^{1} - 1.2 = -1.2$ −ve

$x=2$, $f(2) = 2\log_{10}^{2} - 1.2 = 2(0.3010) - 1.2$ −ve

$= 0.602 - 1.2$

$= -0.598$

$x=3$ $f(3) = 3\log_{10}^{3} - 1.2 = 3(0.4717) - 1.2$ +ve

$= 1.4151 - 1.2$

$= 0.2151$

the roots are 2 and 3

$x_0 = \frac{a+b}{2} = \frac{2+3}{2} = \frac{5}{2} = 2.5$

$f(x) = x\log_{10}^{x} - 1.2$

$f(x) = x \cdot \frac{\log x}{\log_{10}} - 1.2$

$= \frac{x\log x - 1.2\log 10}{\log 10}$

$f'(x) = \frac{[x\frac{1}{x} + \log x \cdot 1 - 0]}{\log 10}$

$f'(x) = \frac{1+\log x}{\log 10}$

By Newton's method

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$= x_0 - \frac{\left(\frac{x_0\log x_0 - 1.2\log 10}{\log 10}\right)}{\frac{1+\log x_0}{\log 10}}$

$= x_0 - \frac{(x_0\log x_0 - 1.2\log 10)}{1+\log x_0}$

$= \frac{x_0(1+\log x_0) - (x_0\log x_0 - 1.2\log 10)}{1+\log x_0}$

$= \frac{x_0 + \cancel{x_0\log x_0} - \cancel{x_0\log x_0} + 1.2\log 10}{1+\log x_0}$

$x_1 = \frac{x_0 + 1.2\log 10}{1+\log x_0}$

$= \frac{(2.5) + 1.2\log 10}{1+\log(2.5)} = \frac{2.5+1.2(1)}{1+0.397940008}$

$= \frac{3.7}{1.397940008} =$

$= 2.646751634$

$x_1 = 2.6468$

$$x_2 = \frac{x_1 + 1.2\log 10}{1+\log x_1}$$

$$= \frac{2.6468+1.2}{1+\log(2.6468)}$$

$$= \frac{3.8468}{1+0.422721126}$$

$$= \frac{3.8468}{1.422721126}$$

$= 2.703832768$

$x_2 = 2.7038$

$$x_3 = \frac{x_2+1.2\log 10}{1+\log x_2}$$

$$= \frac{2.7038+1.2}{1+\log(2.7038)}$$

$$= \frac{3.9038}{1+0.431974563}$$

$$= \frac{3.9038}{1.431974563}$$

$= 2.72383961$

$$= \frac{5.263102112}{1+\log(2.5)}$$

$$= \frac{5.263102112}{1.916290731}$$

$= 2.74650502$

$x_1 = 2.7465$

$$x_2 = \frac{x_1+1.2\log 10}{1+\log x_1}$$

$$= \frac{2.7465+1.2(2.302585093)}{1+\log(2.7465)}$$

$$= \frac{5.509602112}{2.010327374}$$

$= 2.740649201$

$x_2 = 2.7407$

$$x_3 = \frac{x_2+1.2\log 10}{1+\log x_2}$$

$$= \frac{2.7407+1.2(2.302585093)}{1+\log(2.7407)}$$

$$= \frac{5.503802112}{1+1.008213362}$$

$$= \frac{5.503802112}{2.008213362}$$

$= 2.740646097$

$x_3 = 2.7407$

The approximate value $x_2 = x_3 = 2.7407$

7. compute one positive root of $2x - \log_{10}^{x} = 7$

solu) Given that

$$f(x) = 2x - \log_{10}^{x} - 7$$

put $x = 0$

$f(0) = 2(0) - \log_{10}^{0} - 7$, $-ve$

$= 0 - 0 - 7$

$= -7$

$f(1) = 2(1) - \log_{10}^{1} - 7$ -ve

$= 2 - 0 - 7$

$= -5$

$f(2) = 2(2) - \log_{10}^{2} - 7$ -ve

$= 4 - 0.3010 - 7$

$= -3.301$

$f(3) = 2(3) - \log_{10}^{3} - 7$ -ve

$= 6 - 0.477121 - 7$

$= -1.4771$

$f(4) = 2(4) - \log_{10}^{4} - 7$ +ve

$= 8 - 0.60205 - 7$

$= 0.39795$

$x_0 = \frac{a+b}{2} = \frac{3+4}{2} = \frac{7}{2} = 3.5$

$f(x) = 2x - \log_{10}^{x} - 7$

$f'(x) = 2x - \frac{\log x}{\log 10} - 7$

$= \frac{2x\log 10 - \log x - 7\log 10}{\log 10}$

$= \frac{2x(\log x 0) - \log x - 7\log 10}{\log 10}$

$f'(x) = \frac{2\left[x\frac{1}{x} + \log x\right] - \log x - 7\log 10}{\log 10}$

$= \frac{2(1+\log x) - \log x - 7\log 10}{\log 10}$

$= 2 + 2\log x]$

$f'(x) = \frac{1}{\log 10}\left[2\log 10 - \frac{1}{x}\right]$

$= \frac{1}{\log 10}\left[\frac{2x\log 10 - 1}{x}\right]$

$$f'(x) = \frac{2x\log 10 - 1}{x\log 10}$$

By Newton's Iterative method

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= x_0 - \frac{\left[\frac{2x_0\log 10 - \log x_0 - 7\log 10}{\log 10}\right]}{\left[\frac{2x_0\log 10 - 1}{x_0\log 10}\right]}$$

$$= x_0 - \frac{(2x_0\log 10 - \log x_0 - 7\log 10)}{2x_0\log 10 - 1} x_0$$

$$= \frac{x_0(2x_0\log 10 - 1) - (2x_0^2\log 10 - x_0\log x_0 - x_0 7\log 10)}{2x_0\log 10 - 1}$$

$$= \frac{\cancel{2x_0^2\log 10} - x_0 - \cancel{2x_0^2\log 10} + x_0\log x_0 + x_0 7\log 10}{2x_0\log 10 - 1}$$

$$x_1 = \frac{x_0[-1 + \log x_0 + 7\log 10]}{(2x_0\log 10 - 1)}$$

$\ln(3.5) = 1.252762968$

$$x_1 = 3.5\frac{[-1 + \log(3.5) + 7\log 10]}{2(3.5)\log 10 - 1}$$

$$= 3.5\frac{[-1 + 1.252762968 + 7(2.302585093)]}{7(2.302585093) - 1}$$

$$= \frac{3.5[16.37085862]}{15.11809565}$$

$$= \frac{57.29800517}{15.11809565}$$

$$= 3.790027957$$

$$= 3.7900$$

$$x_2 = \frac{x_1[-1+\log x_1 + 7\log_{10}]}{2x_1\log 10 - 1}$$

$$= \frac{3.7900[-1+\log(3.7900)+16.118095655]}{2(3.7900)\log 10 - 1}$$

$$= \frac{3.7900[-1+1.332366019+16.11809565]}{2(3.7900)(2.302585093)-1}$$

$$= \frac{(16.45046167)\,3.7900}{17.71535\,95-1}$$

$$= \frac{62.34724973}{16.453595}$$

$$= 3.789278254$$

$x_2 = 3.7893$

$$x_3 = \frac{x_2[-1+\log x_2 + 7\log 10]}{2x_2\log 10 - 1}$$

$$= \frac{(3.7893)[-1+\log(3.7893)+16.11809565]}{2(3.7893)(2.302585093)-1}$$

$$= \frac{3.7893[-1+1.332181305)+16.11809565)}{17.45037139-1}$$

$$= \frac{3.7893(16.45027696)}{16.45037139}$$

$$= \frac{62.33503447}{16.45037139}$$

$$= 3.789278247$$

$x_3 = 3.7893$

The approximate value $x_2 = x_3 = 3.7893$.

Date 18/8/18

Regula - falsi Method (or) False position Method.

Consider

$y = f(x)$ be the given curve and the given curve passing through $A(x_1, y_1)$ & $B(x_2, y_2)$ then

$$y_1 = f(x_1) \; \& \; y_2 = f(x_2)$$

Then the equation to the curve is

$$y - y_1 = m(x - x_1) \quad \text{where} \quad m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$$

Since the ~~$0 - y_1$~~ = given curve intersect at x-axis so $y = 0$

$$\therefore \quad 0 - y_1 = \left[\frac{y_2 - y_1}{x_2 - x_1}\right](x - x_1)$$

$$x - x_1 = \frac{-y_1 (x_2 - x_1)}{y_2 - y_1}$$

$$x = x_1 - \frac{(x_2 - x_1)\, y_1}{y_2 - y_1}$$

$$x = x_1 - \left[\frac{x_2 - x_1}{f(x_2) - f(x_1)}\right] f(x_1)$$

$$= \frac{x_1 (f(x_2) - f(x_1)) - (x_2 - x_1) f(x_1)}{f(x_2) - f(x_1)}$$

$$= \frac{x_1 f(x_2) - \cancel{x_1 f(x_1)} - x_2 f(x_1) + \cancel{x_1 f(x_1)}}{f(x_2) - f(x_1)}$$

$$x = \frac{x_1 f(x_2) - x_2 f(x_1)}{f(x_2) - f(x_1)}$$

$\therefore$ If $x = x_3$

$$x_3 = \frac{x_1 f(x_2) - x_2 f(x_1)}{f(x_2) - f(x_1)}$$

similarly $x_4 = \dfrac{x_2 f(x_3) - x_3 f(x_2)}{f(x_3) - f(x_2)}$

$$x_5 = \frac{x_3 f(x_4) - x_4 f(x_3)}{f(x_4) - f(x_3)}$$

Date 31/8/18

1. find a real root of the equation $x\log_{10}^{x} - 1.2 = 0$ by False position method

solu| $x\log_{10}^{x} - 1.2 = f(x)$

$$f(x) = x\log_{10}^{x} - 1.2$$

$x=0$, $f(0) = 0\log_{10}^{0} - 1.2 = -1.2$

$x=1$, $f(1) = 1\log_{10}^{1} - 1.2 = 0 - 1.2 = -1.2$

$x=2$, $f(2) = 2\boxed{\log_{10}^{2}} - 1.2$

$= 2(0.3010) - 1.2$

$= 0.6030 - 1.2$

$= -0.5980$

$x=3$, $f(3) = 3\log_{10}^{3} - 1.2$

$= 3(0.477121254) - 1.2$

$= 1.431363764 - 1.2$

$= 0.231363764$

$= 0.2314$

$x_1 = 2$; $f(x_1) = -0.598$

$x_2 = 3$; $f(x_2) = 0.2314$

$$x_3 = \frac{x_1 f(x_2) - x_2 f(x_1)}{f(x_2) - f(x_1)}$$

$$= \frac{2(0.2314) - 3(-0.598)}{0.2314 - (-0.598)}$$

$$= \frac{0.4628 + 1.794}{0.8294}$$

$$= \frac{2.2568}{0.8294}$$

$$= 2.721003135$$

$$x_3 = 2.721$$

$$f(x_3) = 2.721 \log_{10}(2.721) - 1.2$$

$$= 2.721(0.434728541) - 1.2$$

$$= 1.182896362 - 1.2$$

$$= -0.017103637$$

$$= -0.0171$$

$$x_4 = \frac{x_2 f(x_3) - x_3 f(x_2)}{f(x_3) - f(x_2)}$$

$$= \frac{3(-0.0171) - 2.721(0.2314)}{-0.0171 - 0.2314}$$

$$= \frac{-0.0513 - 0.6296394}{-0.2485}$$

$$= \frac{-0.6809394}{-0.2485}$$

$$= 2.740198793$$

$$x_4 = 2.7402$$

$$f(x_4) = 2.7402 \log_{10}(2.7402) - 1.2$$

$$= 2.7402(0.437782262) - 1.2$$

$$= 1.199610954 - 1.2$$

$$= -0.000389046$$

$$= -0.0004$$

$$x_5 = \frac{x_3 f(x_4) - x_4 f(x_3)}{f(x_4) - f(x_3)}$$

$$= \frac{2.721 \times -0.0024 - 2.7402 \times (-0.0171)}{-0.0024 - (-0.0171)}$$

$$= \frac{-0.0010884 + 0.046857 42}{0.0167}$$

$$= \frac{0.04576902}{0.0167}$$

$$= 2.74065988$$

$$x_5 = 2.7407$$

$$f(x_5) = 2.7407 \log_{10}(2.7407) - 1.2$$

$$= 2.7407(0.437861499) - 1.2$$

$$= 1.200047012 - 1.2$$

$$= 0.000047012488$$

$$= 0.0001$$

$$x_6 = \frac{x_4 f(x_5) - x_5 f(x_4)}{f(x_5) - f(x_4)}$$

$$= \frac{2.7402 \times 0.0001 - 2.7407(-0.0004)}{0.0001 - (-0.0004)}$$

$$= \frac{0.00027402 + 0.00109628}{0.0005}$$

$$= \frac{0.0013703}{0.0005}$$

$$= 2.7406$$

$$f(x_6) = 2.7406 \log(2.7406) - 1.2$$

$$= 2.7406(0.437845653) - 1.2$$

$$= 1.199959798 - 1.2$$

$= -0.0000402023171$

$= -0.0$

$x_7 = x_6 = 2.7406$

The roots of the equation

$x_7 = x_6 = 2.7406$

2. Find the real roots of the equation

$x - e^{-x} = 0$

Solu)

$x - e^{-x} = 0$

$f(x) = x - e^{-x}$

$x = 0,\ f(0) = 0 - e^{-0} = -1$

$x = 1,\ f(1) = 1 - e^{-1} = 0.6321205588$

$= 0.6321$ 0.54

$x_1 = 0,\ f(x_1) = -1$

$x_2 = 1,\ f(x_2) = 0.6321$

$$x_3 = \frac{x_1 f(x_2) - x_2 f(x_1)}{f(x_2) - f(x_1)}$$

$$= \frac{0(0.6321) - (1)(-1)}{0.6321 - (-1)}$$

$$= \frac{0+1}{0.6321+1} = \frac{1}{1.6321} = 0.612707554$$

$= 0.6127$

$f(x_3) = (0.6127)\log_{10}(0.6127) - \cancel{1.2}\ e^{-0.6127}$

$[= (0.6127)(-0.212752119) - \cancel{1.2}\ 0.5418858$

$= -0.130353223 - \cancel{1.2}\ 0.5418858$

$= -\cancel{1}.330353224]$ 0.07081419954

$= 0.0708$

$$x_4 = \frac{x_2 f(x_3) - x_3 f(x_2)}{f(x_3) - f(x_2)}$$

$$= \frac{1 \times 0.0708 - 0.6127 \times 0.6321}{0.0708 - 0.6321}$$

$$= \frac{0.0708 - 0.38728767}{-0.5613}$$

$$= \frac{-0.31648767}{-0.5613} = 0.563847621$$

$$x_4 = 0.5639$$

$$f(x_4) = 0.5639 \log_{10}(0.5639) - e^{-0.5639}$$

$$= 0.5639(-0.248797905) - (0.568985686)$$

$$= 0.140297138 - 0.568985686$$

$$= -0.00508568 60$$

$$f(x_4) = -0.0051$$

$$x_5 = \frac{x_3 f(x_4) - x_4 (f(x_3))}{f(x_4) - f(x_3)}$$

$$= \frac{0.6127 \times -0.0051 - 0.5639 \cdot (0.0708)}{-0.0051 - 0.0708}$$

$$= \frac{-0.04304889}{-0.0759} = 0.567179051$$

$$x_5 = 0.5672$$

$$f(x_5) = 0.5672 - e^{-0.5672}$$

$$= 0.5672 - 0.567111128$$

$$= 0.0000887114156$$

$$= 0.0001$$

$$x_6 = \frac{x_4 f(x_5) - x_5 f(x_4)}{f(x_5) - f(x_4)}$$

$$= \frac{0.5639 \times 0.0001 - 0.56721 \times -0.0051}{0.0001 - (-0.0051)}$$

$$= \frac{0.00294911}{0.0052}$$

$$= 0.567136538$$

$x_6 = 0.5671$

$f(x_6) = 0.5671 - e^{-0.5671}$

$= 0.5671 - 0.567167842$

$= -0.0000067842$

$= +0.0009$

$$x_7 = \frac{x_5 f(x_6) - x_6 f(x_5)}{f(x_6) - f(x_5)}$$

$$= \frac{0.5672(0.0001) - 0.5671(0.5672)}{0.0001 - 0.5672}$$

$$= \frac{0.00005672 - 0.32165912}{-0.5671}$$

$$= \frac{-0.3216024}{-0.5671}$$

$= 0.567099982$

$= 0.5671$

$x_6 = x_7 = 0.5671$

The real roots are $x_6 = x_7 = 0.5671$

3. $x^3 - 5x + 3 = 0$ by using false position method c

solu) Given that

$0 = x^3 - 5x + 3 = f(x)$

$x = 0, \; f(0) = 0 - 5(0) + 3 = 3$

$x = 1, \; f(1) = 1 - 5(1) + 3 = -1$

$x = 2, \; f(2) = 2^3 - 5(2) + 3$

$= 8 - 10 + 3$

$x_1 = 1, \quad f(x_1) = -1$

$x_2 = 2, \quad f(x_2) = 1$

$$x_3 = \frac{x_1 f(x_2) - x_2 f(x_1)}{f(x_2) - f(x_1)}$$

$$= \frac{1(1) - 2(-1)}{1-(-1)} = \frac{1+2}{1+1} = \frac{3}{2} = 1.5$$

$x_3 = 1.5$

$$f(x_3) = (1.5)^3 - 5(1.5) + 3$$

$$= 3.375 - 7.5 + 3$$

$$= -1.125$$

$$x_4 = \frac{x_2 f(x_3) - x_3 f(x_2)}{f(x_3) - f(x_2)}$$

$$= \frac{2(-1.125) - (1.5)(1)}{-1.125 - 1}$$

$$= \frac{-2.25 - 1.5}{-2.125}$$

$$= \frac{-3.75}{-2.125} = 1.764705882$$

$x_4 = 1.765$

$$f(x_4) = (1.765)^3 - 5(1.765) + 3$$

$$= 5.498372125 - 8.825 + 3$$

$$= -0.326627875$$

$f(x_4) = -0.327$

$$x_5 = \frac{x_3 f(x_4) - x_4 f(x_3)}{f(x_4) - f(x_3)}$$

$$= \frac{(1.5)(-0.327) - (1.765)(-1.125)}{-0.327 - (-1.125)}$$

$$= \frac{-0.4905+1.985625}{0.798}$$

$$= \frac{1.495125}{0.798}$$

$$= 1.873590226$$

$$= 1.874$$

$$f(x_5) = (1.874)^3 - 5(1.874)+3$$

$$= 6.581255624 - 9.37 + 3$$

$$= 0.211255624$$

$$= 0.211$$

$$x_6 = \frac{x_4 f(x_5) - x_5 f(x_4)}{f(x_5) - f(x_4)}$$

$$= \frac{1.765(0.211) - (1.874)(-0.327)}{0.211 - (-0.327)}$$

$$= \frac{0.372415 + 0.612798}{0.538}$$

$$= \frac{0.985213}{0.538}$$

$$= 1.831250929$$

$$x_6 = 1.831$$

$$f(x_6) = (1.831)^3 - 5(1.831)+3$$

$$= 6.138539191 - 9.155 + 3$$

$$= -0.01646 0809$$

$$= -0.016$$

$$x_7 = \frac{x_5 f(x_6) - x_6 f(x_5)}{f(x_6) - f(x_5)}$$

$$= \frac{1.874(-0.016) - 1.831(0.211)}{-0.016 - 0.211}$$

$$= \frac{-0.029984 - 0.386341}{-0.227}$$

$$= \frac{-0.416325}{-0.227}$$

$$= +1.834030837$$

$$= 1.834$$

$$f(x_8) = (1.834)^3 - 5(1.834) + 3$$

$$= 6.168761704 - 9.17 + 3$$

$$= -0.001238296$$

$$= -0.001$$

$$x_8 = \frac{x_6 f(x_7) - x_7 f(x_6)}{f(x_7) - f(x_6)}$$

$$= \frac{1.831(-0.001) - (1.834)(-0.016)}{-0.001 - (-0.016)}$$

$$= \frac{-0.001831 + 0.029344}{0.015}$$

$$= \frac{0.027513}{0.015} = 1.8342 = 1.834$$

$f(x_8) =$ $x_7 = x_8 = 1.834$

The real roots are $x_7 = x_8 = 1.834$

Date 1/9/18

4 Find the real root of the Equation $\tan x + \tanh x = 0$ in the introval $[1.6, 3)$

solu) Given

$$f(x) = \tan x + \tanh x$$

$$\therefore f(1.6) = \tan(1.6) + \tanh(1.6)$$

$$= -34.23253274 + 0.921668554$$

$$= -33.31086418$$

$$f(2) = \tan 2 + \tanh 2$$
$$= -2.185039863 + 0.96402758$$
$$= -1.221012283$$

$$f(2.2) = \tan(2.2) + \tanh(2.2)$$
$$= -1.373823057 + 0.97574313$$
$$= -0.398079926$$

$$f(2.4) = \tan(2.4) + \tanh(2.4)$$
$$= -0.916014289 + 0.983674857$$
$$= 0.067660568$$

$x_1 = 2.2 \quad f(x_1) = -0.3981$

$x_2 = 2.4 \quad f(x_2) = 0.0677$

$$x_3 = \frac{x_1 f(x_2) - x_2 f(x_1)}{f(x_2) - f(x_1)}$$

$$= \frac{(2.2)(0.0677) - (2.4)(-0.3981)}{0.0677 - (-0.3981)}$$

$$= \frac{(2.2)(0.0677) + (2.4)(0.3981)}{0.0677 + 0.3981}$$

$$= \frac{0.14894 + 0.95544}{0.4658} = \frac{1.10438}{0.4658}$$

$$= 2.37093173$$

$$x_3 = 2.3709$$

$$f(x_3) = \tan(2.3709) + \tanh(2.3709)$$
$$= -0.971013157 + 0.982705001$$
$$= 0.01169184$$
$$= 0.0117$$

$$x_4 = \frac{x_2 f(x_3) - x_3 f(x_2)}{f(x_3) - f(x_2)}$$

$$= \frac{(2.4)(0.0117) - 2.3709 \times 0.0677}{0.0117 - 0.0677}$$

$$= \frac{0.02808 - 0.16050993}{-0.056}$$

$$= \frac{-0.13242993}{-0.056} = 2.364820179$$

$x_4 = 2.3648 = 2.3645$

$f(x_4) = \tan(2.3648) + \tanh(2.3648)$

$= -0.982935408 + 0.982494568$

$= -0.0004408403$

$= -0.0004$

$$x_5 = \frac{x_3 f(x_4) - x_4 f(x_3)}{f(x_4) - f(x_3)}$$

$$= \frac{(2.3709)(-0.0004) - (2.3648)(0.0117)}{-0.0004 - 0.0117}$$

$$= \frac{-0.00094836 - 0.02766816}{-0.0121}$$

$$= \frac{-0.02861652}{-0.0121}$$

$= 2.365001653$

$x_5 = 2.365$

$f(x_5) = \tan(2.365) + \tanh(2.365)$

$= -0.982542253 + 0.982501507$

$= \underline{0.00004074} = 0.0000$

$$x_6 = \frac{x_4 f(x_5) - x_5 f(x_4)}{f(x_5) - f(x_4)}$$

$$= \frac{(2.3648)(0.0000) - (2.365)(-0.0004)}{0.0000 - (-0.0004)}$$

$$= \frac{0.000946}{0.0004}$$

$$= 2.365$$

$x_5 = x_6 = 2.365$ are real roots.

5 Find the root of the given Equation $xe^x = \cos x$ in intrval $(0,1)$

solu] Given $f(x) = xe^x - \cos x$

$x = 0.5$. $f(0.5) = (0.5)e^{0.5} - \cos(0.5)$

$= (0.5)(1.648721271) - 0.877582561$

$= 0.824360635 - 0.877582561$

$= -0.053221926$

$x = 0.6$ $f(0.6) = (0.6)e^{0.6} - \cos(0.6)$

$= (0.6)(1.8221188) - 0.825335614$

$= 1.09327128 - 0.825335614$

$= 0.267935666$

$x_1 = 0.5$, $f(x_1) = -0.0532$

$x_2 = 0.6$ $f(x_2) = 0.2679$

$$x_3 = \frac{x_1 f(x_2) - x_2 f(x_1)}{f(x_2) - f(x_1)}$$

$$= \frac{(0.5)(0.2679) - (0.6)(-0.0532)}{0.2679 - (-0.0532)}$$

$$= \frac{0.13395 + 0.03192}{0.3211}$$

$$= \frac{0.16587}{0.3211} = 0.516568047$$

$$= 0.5166$$

$f(x_3) = (0.5166)e^{0.5166} - \cos(0.5166)$

$= -0.003517432952$

$= -0.0035$

$$x_4 = \frac{x_2 f(x_3) - x_3 f(x_2)}{f(x_3) - f(x_2)}$$

$$= \frac{(0.6)(-0.0035) - (0.5166)(0.2679)}{-0.0035 - 0.2679}$$

$$= \frac{-0.0021 - 0.13839714}{-0.2714}$$

$$= \frac{-0.14049714}{-0.2714}$$

$$= 0.517675534$$

$$= 0.5177$$

$$f(x_4) = (0.5177)e^{0.5177} - \cos(0.5177)$$

$$= (0.5177)(1.678163432) - 0.868959707$$

$$= 0.868785208 - 0.868959707$$

$$= -0.0002$$

$$x_5 = \frac{x_3 f(x_4) - x_4 f(x_3)}{f(x_4) - f(x_3)}$$

$$= \frac{(0.5166)(-0.0002) - (0.5177)(-0.0035)}{-0.0002 + 0.0035}$$

$$= \frac{-0.00010332 + 0.00181195}{0.0033}$$

$$= \frac{0.00170863}{0.0033} = 0.51776666$$

$$= 0.5178$$

$$f(x_5) = (0.5178)e^{0.5178} - \cos(0.5178)$$

$x^3 - 4x + 1$

$$= (0.5178)(1.678331256) - 0.868910215$$

$$= 0.869039924 - 0.868910215$$

$$= 0.00012970958 28$$

$$= 0.0001$$

$$x_6 = \frac{x_4 f(x_5) - x_5 f(x_4)}{f(x_5) - f(x_4)}$$

$$= \frac{(0.5177)(0.0001) - (0.5178)(-0.0002)}{0.0001 - (-0.0002)}$$

$$= \frac{0.00005177 + 0.00010356}{0.0003}$$

$$= \frac{0.00015533}{0.0003}$$

$$= 0.51776666$$

$$= 0.5178$$

$x_5 = x_6 = 0.5178$

6. $x^3 - 4x + 1$ 7. $xe^x = 3$

Solu) Given that

$f(x) = x^3 - 4x + 1$

$x = 0$, $f(0) = 0 - 4(0) + 1 = 1$

$x = 1$, $f(1) = 1 - 4(1) + 1 = -2$

$x = 2$, $f(2) = 2^3 - 4(2) + 1$

$= 8 - 8 + 1 = 1$

$x_1 = 1$, $f(x_1) = -2$

$x_2 = 2$, $f(x_2) = 1$

$$x_3 = \frac{x_1 f(x_2) - x_2 f(x_1)}{f(x_2) - f(x_1)}$$

$$= \frac{1(1) - 2(-2)}{1 - (-2)} = \frac{1+4}{1+2} = \frac{5}{3}.$$

$= 1.66666667$

$x_3 = 1.6667$

$f(x_3) = (1.6667)^3 - 4(1.6667) + 1$

$= 4.629907413 - 6.6668 + 1$

$= -1.036892587$

$$= -1.03609$$

$$x_4 = \frac{x_2 f(x_3) - x_3 f(x_2)}{f(x_3) - f(x_2)}$$

$$= \frac{2(-1.0369) - 1.6667(1)}{-1.0369 - 1}$$

$$= \frac{-2.0738 - 1.6667}{-2.0369} = \frac{-3.7405}{-2.0369}$$

$$= 1.836368992$$

$$x_4 = 1.8364$$

$$f(x_4) = (1.8364)^3 - 4(1.8364) + 1$$

$$= 6.193011013 - 7.3456 + 1$$

$$= -0.152588987$$

$$= -0.1526$$

$$x_5 = \frac{x_3 f(x_4) - x_4 f(x_3)}{f(x_4) - f(x_3)}$$

$$= \frac{1.6667(-0.1526) - (1.8364)(-1.0369)}{-0.1526 + 1.0369}$$

$$= \frac{-0.25433842 + 1.90416316}{0.8843}$$

$$= \frac{1.64982474}{0.8843} = 1.865684428$$

$$x_5 = 1.8657$$

$$f(x_5) = (1.8657)^3 - 4(1.8657) + 1$$

$$= 6.494196639 - 7.4628 + 1$$

$$= 0.031396639$$

$$= 0.0314$$

$$x_6 = \frac{x_4 f(x_5) - x_5 f(x_4)}{f(x_5) - f(x_4)}$$

$$= \frac{1.8364(0.0314) - 1.8657(-0.1526)}{0.0314 + 0.1526}$$

$$= \frac{0.05766296 + 0.28470582}{0.184}$$

$$= \frac{0.34236878}{0.184}$$

$$= 1.860699891$$

$$x_6 = 1.8607$$

$$f(x_6) = (1.8607)^3 - 4(1.8607) + 1$$

$$= 6.442123895 - 7.4428 + 1$$

$$= -0.000676105$$

$$= -0.0007$$

$$x_7 = \frac{x_5 f(x_6) - x_6 f(x_5)}{f(x_6) - f(x_5)}$$

$$= \frac{1.8657(-0.0007) - (1.8607)(0.0314)}{-0.0007 - 0.0314}$$

$$= \frac{-0.00130599 - 0.05842598}{-0.0321}$$

$$= \frac{-0.05973197}{-0.0321}$$

$$= 1.860809034$$

$$x_7 = 1.8608$$

$$f(x_7) = (1.8608)^3 - 4(1.8608) + 1$$

$$= 6.443162612 - 7.4432 + 1$$

$$= -0.00037388$$

$$x_8 = \frac{x_6 f(x_7) - x_7 f(x_6)}{f(x_7) - f(x_6)}$$

$$= \frac{1.8607(-0.000) - (1.8608)(-0.0007)}{-0.000 + 0.0007}$$

$$= \frac{0 + 0.00130256}{0.0007}$$

$x_8 = 1.8608$

$x_7 = x_8 = 1.8608$ the real roots

7 Given that

$f(x) = xe^x - 3$

$x=0,\ f(0) = 0e^0 - 3$
$= -3$

$x=1,\ f(1) = 1e^1 - 3$
$= 2.718281828 - 3$
$= -0.281718171$
$= -0.2817$

$x=2,\ f(2) = 2e^2 - 3$
$= 2(7.389056099) - 3$
$= 14.7781122 - 3$
$= 11.7781122$
$= 11.7781$

$x_1 = 1,\ f(x_1) = -0.2817$
$x_2 = 2,\ f(x_2) = 11.7781$

$$x_3 = \frac{x_1 f(x_2) - x_2 f(x_1)}{f(x_2) - f(x_1)}$$

$$= \frac{1(11.7781) - 2(-0.2817)}{11.7781 + 0.2817}$$

$$= \frac{11.7781 + 0.5634}{12.0598}$$

$$= \frac{12.3415}{12.0598}$$

$$= 1.023358596$$

$$x_3 = 1.0234$$

$$f(x_3) = (1.0234)e^{1.0234} - 3$$

$$= (1.0234)(2.782639673) - 3$$

$$= 2.847753442 - 3$$

$$= -0.152246558$$

$$= -0.1522$$

$$x_4 = \frac{x_2 f(x_3) - x_3 f(x_2)}{f(x_3) - f(x_2)}$$

$$= \frac{2(-0.1522) - (1.0234)(11.7781)}{-0.1522 - 11.7781}$$

$$= \frac{-0.3044 - 12.05370754}{-11.9303}$$

$$= \frac{-12.35810754}{-11.9303}$$

$$= +1.035858909$$

$$= 1.0359$$

$$f(x_4) = (1.0359)e^{1.0359} - 3$$

$$= (1.0359)(2.817640972) - 3$$

$$= 2.918794283 - 3$$

$$= -0.0812205717$$

$$= -0.0812$$

$$x_5 = \frac{x_3 f(x_4) - x_4 f(x_3)}{f(x_4) - f(x_3)}$$

$$= \frac{(1.0234)(-0.0812) - (1.0359)(-0.1522)}{-0.0812 + 0.1522}$$

$$= \frac{-0.08310008 + 0.15766398}{0.071}$$

$$= \frac{0.0745639}{0.071}$$

$$= \cancel{0.0745639} \ 1.050195775$$

$$= \cancel{0.0746} \ 1.0502$$

$$f(x_5) = (1.0502)e^{1.0502} - 3$$

$$= (1.0502)(2.858222705) - 3$$

$$= 3.001705485 - 3$$

$$= 0.001705485257$$

$$= 0.0017$$

$$x_6 = \frac{x_4 f(x_5) - x_5 f(x_4)}{f(x_5) - f(x_4)}$$

$$= \frac{1.0359(0.0017) - (1.0501)(-0.0812)}{0.0017 + 0.0812}$$

$$= \frac{0.00176103 + 0.08526812}{0.0829}$$

$$= \frac{0.08702915}{0.0829}$$

$$= 1.049808806$$

$$= 1.0498$$

$$f(x_6) = (1.0498)e^{1.0498} - 3$$

$$= (1.0498)(2.857079645) - 3$$

$$= 2.999362211 - 3$$

$$= -0.000637788\,6908$$

$= -0.0006$

$$x_7 = \frac{x_5 f(x_6) - x_6 f(x_5)}{f(x_6) - f(x_5)}$$

$$= \frac{(1.0501)(-0.0006) - (1.0498)(0.0017)}{-0.0006 - 0.0017}$$

$$= \frac{0.00063006 - 0.00178466}{-0.0023}$$

$$= \frac{-0.0011546}{-0.0023}$$

$= 0.502$

$f(x_7) = (0.502) e^{0.502} - 3$

$= (0.502)(1.652022013) - 3$

$= 0.82931505 - 3$

$= -2.170684949$

$= -2.1707$

$$x_8 = \frac{x_6 f(x_7) - x_7 f(x_6)}{f(x_7) - f(x_6)}$$

$$= \frac{1.0498(-2.1707) - (0.502)(-0.0006)}{-2.1707 + 0.0006}$$

$$= \frac{-2.27880086 + 0.0003012}{-2.1701}$$

$$= \frac{-2.27849966}{-2.1701}$$

$= +1.049951458$

$= 1.0491$

$f(x_8) = (1.0491) e^{1.0491} - 3$

$= (1.0491)(2.855080358) - 3$

$= 2.995264836 - 3$

$= -0.004735163839$

$= -0.0047$

$$x_9 = \frac{x_7 f(x_8) - x_8 f(x_7)}{f(x_8) - f(x_7)}$$

$$= \frac{(0.502)(-0.0047) - (1.0491)(-2.1707)}{-0.0047 + 2.1707}$$

$$= \frac{-0.0023594 + 2.27728137}{2.166}$$

$$= \frac{2.27492197}{2.166}$$

$= 1.050287151$

$= 1.0503$

$f(x_9) = (1.0503) e^{1.0503} - 3$

$= (1.0503)(2.858508542) - 3$

$= 3.002291522 - 3$

$= 0.002291521669$

$= 0.0023$

$$x_{10} = \frac{x_8 f(x_9) - x_9 f(x_8)}{f(x_9) - f(x_8)}$$

$$= \frac{(1.0491)(0.0023) - (1.0503)(-0.0047)}{0.0023 + 0.0047}$$

$$= \frac{0.00241293 + 0.00493641}{0.007}$$

$$= \frac{0.00734934}{0.007}$$

$$= 1.049905714$$

$$= 1.0499$$

$$f(x_{10}) = (1.0499)e^{1.0499} - 3$$

$$= (1.0499)(2.857365367) - 3$$

$$= 2.999947899 - 3$$

$$= 0.00005210093563$$

$$= 0.0000$$

$$x_{11} = \frac{x_9 f(x_{10}) - x_{10} f(x_9)}{f(x_{10}) - f(x_9)}$$

$$= \frac{(1.0503)(0.0000) - (1.0499)(0.0023)}{0.0000 - 0.0023}$$

$$= \frac{-0.00241477}{-0.0023}$$

$$= 1.0499$$

$x_{10} = x_{11} = 1.0499$ real values.

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} k \\ -4k \\ k \end{bmatrix} = k \begin{bmatrix} 1 \\ -4 \\ 1 \end{bmatrix}$$

Date 13/12/18 **Gauss - Seidl Iteration Method**

We will consider the system of equations

$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$; $a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2$;

$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3$; $\rightarrow$ (1)

where the diagonal co-efficients are not zero and are large comparae to other co-efficients such a system is called diagonally dominant system

1. Solve $10x + y + z = 12$; $2x + 10y + z = 13$; $2x + 2y + 10z = 14$ by Gauss-seidl iteration method

Solu) Given equations

$$\left.\begin{aligned} 10x + y + z &= 12 \\ 2x + 10y + z &= 13 \\ 2x + 2y + 10z &= 14 \end{aligned}\right\} \rightarrow \text{(I)}$$

Equation (I) is a diagonally dominent system

$$10x + y + z = 12$$

$$x = (12 - y - z)\frac{1}{10} \rightarrow (1)$$

$$2x + 10y + z = 13$$

$$y = \frac{1}{10}(13 - 2x - z) \rightarrow (2)$$

$$2x + 2y + 10z = 14$$

$$z = \frac{1}{10}(14 - 2x - 2y) \rightarrow (3)$$

Put $y=0,\ z=0$ in eq ①

$$x^{(1)} = \frac{1}{10}(12-0-0) = 1.2$$

Put $x=1.2$, $z=0$ in eq ②

$$y^{(1)} = \frac{1}{10}(13-2(1.2)-0)$$

$$y^{(1)} = 1.06$$

Put $x=1.2$, $y=1.06$ in eq ③

$$z^{(1)} = \frac{1}{10}(14-2(1.2)-2\cdot(1.06))$$

$$z^{(1)} = 0.948$$

$$x^{(1)} = 1.2,\ y^{(1)} = 1.06,\ z^{(1)} = 0.948$$

II - Iteration

Put $y=1.06$, $z=0.948$ in eq ①

$$x^{(2)} = \frac{1}{10}(12-1.06-0.948)$$

$$= 0.9992$$

Put $x=0.9992$, $z=0.948$ in eq ②

$$y^{(2)} = \frac{1}{10}(13-2(0.9992)-0.948)$$

$$= 1.00517$$

Put $x=0.9992$; $y=1.00517$ in eq ③

$$z^{(2)} = \frac{1}{10}(14-2(0.9992)-2(1.00517))$$

$$= 0.99978 = 0.9991$$

$$x^{(2)} = 0.9992;\ y^{(2)} = 1.0054;\ z^{(2)} = 0.9998$$

III - Iteration

Put $y=1.0054$, $z=0.9998$ in eq ①

$$x^{(3)} = \frac{1}{10}(12-1.0054-0.9998)$$

$= 0.99955$

put $x = 0.99955$; $z = 0.9991$ in eq ②

$$y^{(3)} = \frac{1}{10}(13 - 2(0.9992) - 0.9991)$$

$$= 1.0001$$

put $x = 0.9995$; $y = 1.0001$ in eq ③

$$z^{(3)} = \frac{1}{10}(14 - 2(0.99955) - 2(1.0001))$$

$z^{(3)} = 1.0001$; $x^{(3)} = 0.99955$; $y^{(3)} = 1.0001$

$z^{(3)} = 1.0001$

IV - Iteration

put $z^{(3)} = 1.0001$, $y^{(3)} = 1.0001 \rightarrow$ ①

$$x = \frac{1}{10}(12 - 1.0001 - 1.0001)$$

$$x^{(4)} = 0.99998$$

put $x^{(4)} = 1$, in eq ② ; $z = 1.0001$

$$y = \frac{1}{10}(13 - 2(1) - 1.0001)$$

$$y^{(4)} = 0.999 = 1$$

put $x = 1$, $y = 0.99 = 1$ in eq ③

$$z = \frac{1}{10}(14 - 2(1) - 2(1))$$

$$z^{(4)} = 1$$

$x^{(4)} = 0.99998$, $y^{(4)} = 0.999 = 1$; $z^{(4)} = 1$

we have

Variable	I st Approximation	2nd	3rd	4th
x	1.2	0.9992	0.99955	1
y	1.06	1.0054	1.0001	1
z	0.948	0.9991	1.0001	1

H.W
2. Solve

$27x+6y-z=85$
$6x+15y+2z=72$
$x+y+54z=110$

3. $8x_1-3x_2+2x_3=20$
$4x_1+11x_2-x_3=33$
$6x_1+3x_2+12x_3=36$

4. $x+10y+z=6$
$10x+y+z=6$
$x+y+10z=6$

Solu] 2) Given equations

$$\left.\begin{aligned}27x+6y-z&=85\\6x+15y+2z&=72\\x+y+54z&=110\end{aligned}\right\}\rightarrow (A)$$

Equation (A) is a diagonally dominant system.

$27x+6y-z=85$

$$x=(85-6y+z)\frac{1}{27}\rightarrow (1)$$

$6x+15y+2z=72$

$$y=(72-6x-2z)\frac{1}{15}\rightarrow (2)$$

$x+y+54z=110$

$$z=(110-x-y)\frac{1}{54}\rightarrow (3)$$

$\Rightarrow$ put $y=0,\ z=0$ in eq (1)

$$x^{(1)}=(85-0+0)\frac{1}{27}$$

$$x^{(1)}=3.14815$$

$\Rightarrow$ put $x=3.14815;\ z=0$ in eq (2)

$$y^{(1)}=(72-6(3.14815)-2(0))\frac{1}{15}$$

$$y^{(1)}=3.54074$$

$\Rightarrow$ put $x=3.14815;\ y=3.54074$ in eq (3)

$$z^{(1)} = (110 - 3.14815 - 3.54074)\frac{1}{54}$$

$$z^{(1)} = 1.9135$$

$\therefore x^{(1)} = 3.14815 ; \ y^{(1)} = 3.54074 ; \ z^{(1)} = 1.9135$

* II - Iteration

$\Rightarrow$ put $[x = 3.14815]$; $z = 1.9135$; $y = 3.54074$ in ①

$$x^{(2)} = (85 - 6(3.54074) + 1.9135)\frac{1}{27}$$

$$x^{(2)} = 2.4322$$

$\Rightarrow$ put $x = 2.4322$; $z = 1.9135$ in eq ②

$$y^{(2)} = (72 - 6(2.4322) - 2(1.9135))\frac{1}{15}$$

$$y(2) = 3.572$$

$\Rightarrow$ put $x = 2.4322$; $y = 3.572$ in eq ③

$$z^{(2)} = (110 - 2.4322 - 3.572)\frac{1}{54}$$

$$z^{(2)} = 1.9258$$

$\therefore x^{(2)} = 2.4322 ; \ y^{(2)} = 3.572 ; \ z^{(2)} = 1.9258$

* III - Iteration

$\Rightarrow$ put $y = 3.572$; $z = 1.9258$ in eq ①

$$x^{(3)} = (85 - 6(3.572) + 1.9258)\frac{1}{27}$$

$$x(3) = 2.4257$$

$\Rightarrow$ put $x = 2.4257$; $z = 1.9258$ in eq ②

$$y^{(3)} = (72 - 6(2.4257) - 2(1.9258))\frac{1}{15}$$

$$y(3) = 3.573$$

$\Rightarrow$ put $x = 2.4257$; $y = 3.573$ in eq ③

$$z^{(3)} = (110 - 2.4257 - 3.573)\frac{1}{54}$$

$$= 1.9258$$

$x^{(3)} = 2.4257$; $y^{(3)} = 3.573$; $z^{(3)} = 1.92595$.

IV - Iteration

$\Rightarrow$ put $(x = 2.4257)$; $y = 3.573$; $z = 1.926$ in eq ①

$$x^{(4)} = (85 - 6(3.573) + 1.926)\frac{1}{27}$$

$$x^{(4)} = 2.4255$$

$\Rightarrow$ put $x = 2.4255$; $z = 1.926$ in eq ②

$$y^{(4)} = (72 - 6(2.4255) - 2(1.926))\frac{1}{15}$$

$$y^{(4)} = 3.573$$

$\Rightarrow$ put $x = 2.4255$; $y = 3.573$; in eq ③

$$z^{(4)} = (110 - 2.4255 - 3.573)\frac{1}{54}$$

$$= 1.92595$$

$$= 1.926$$

$\therefore x^{(4)} = 2.4255$; $y^{(4)} = 3.573$; $z^{(4)} = 1.926$

V - Iteration

$\Rightarrow$ put $y = 3.573$; $z = 1.926$ in eq ①

$$x^{(5)} = (85 - 6(3.573) + 1.926)\frac{1}{27}$$

$$= 2.4255$$

$\Rightarrow$ put $x = 2.4255$; $z = 1.926$ in eq ②

$$y^{(5)} = (72 - 6(2.4255) - 2(1.926))\frac{1}{15}$$

$$= 3.573$$

$\Rightarrow$ put $x = 2.4255$; $y = 3.573$ in eq ③

$$z^{(5)} = (110 - 2.4255 - 3.573)\frac{1}{54}$$

$= 1.926$

$\therefore x^{(5)} = 2.4255 ; y^{(5)} = 3.573 ; z^{(5)} = 1.926$

variable	1st	2nd	3rd	4th	5th
x	3.14815	2.4322	2.4257	2.4255	2.4255
y	3.540751	3.572	3.573	3.573	3.573
z	1.9135	1.9258	1.92595	1.926	1.926

4. Given Equations

$x + 10y + z = 6$

$10x + y + z = 6$

$x + y + 10z = 6$

$$\left.\begin{array}{l} 10x + y + z = 6 \\ x + 10y + z = 6 \\ x + y + 10z = 6 \end{array}\right\} \rightarrow (A)$$

Equation (A) is a diagonally dominant system

$10x + y + z = 6$

$$x = (6 - y - z)\frac{1}{10} \rightarrow (1)$$

$x + 10y + z = 6$

$$y = (6 - x - z)\frac{1}{10} \rightarrow (2)$$

$x + y + 10z = 6$

$$z = (6 - x - y)\frac{1}{10} \rightarrow (3)$$

I. Iteration

$\Rightarrow$ put $y = 0$; $z = 0$ in eq (1)

$$x^{(1)} = (6 - 0 - 0)\frac{1}{10} = 0.6$$

$\Rightarrow$ put $x = 0.6$; $z = 0$ in eq (2)

$$y^{(1)} = (6 - 0.6 - 0)\frac{1}{10}$$

$= 0.54$

$\Rightarrow$ put $x = 0.6$; $y = 0.54$ in eq ③

$$z^{(1)} = (6 - 0.6 - 0.54)\frac{1}{10}$$

$$= 0.486$$

$x^{(1)} = 0.6$; $y^{(1)} = 0.54$; $z^{(1)} = 0.486$

II - Iteration

$\Rightarrow$ put $y = 0.54$; $z = 0.486$ in eq ①

$$x^{(2)} = (6 - 0.54 - 0.486)\frac{1}{10}$$

$$x^{(2)} = 0.4974$$

$\Rightarrow$ put $x = 0.4974$; $z = 0.486$ in eq ②

$$y^{(2)} = (6 - 0.4974 - 0.486)\frac{1}{10}$$

$$= 0.502$$

$\Rightarrow$ put $x = 0.4974$; $y = 0.502$ in eq ③

$$z^{(2)} = (6 - 0.4974 - 0.502)\frac{1}{10}$$

$$z^{(2)} = 0.50006$$

$\therefore x^{(2)} = 0.4974$; $y^{(2)} = 0.502$; $z^{(2)} = 0.50006$

III - Iteration

$\Rightarrow$ put $y = 0.502$; $z = 0.50006$ in eq ①

$$x^{(3)} = (6 - 0.502 - 0.50006)\frac{1}{10}$$

$$= 0.4998$$

$\Rightarrow$ put $x = 0.4998$; $z = 0.50006$ in eq ②

$$y^{(3)} = (6 - 0.4998 - 0.50006)\frac{1}{10}$$

$$= 0.500014$$

$\Rightarrow$ put $x = 0.4998$; $y = 0.500014$ in eq ③

$$z^{(3)} = (6 - 0.4998 - 0.500014)$$

$= 0.500019$

$x^{(3)} = 0.4998$; $y^{(3)} = 0.500014$; $z^{(3)} = 0.500019$

IV - Iteration

$\Rightarrow$ put ($x = 0.4998$) $y = 0.500014$; $z = 0.500019$ in eq ①

$$x^{(4)} = (6 - 0.500014 - 0.500019)\frac{1}{10}$$

$$= 0.49910$$

$\Rightarrow$ put $x = 0.49910$; $z = 0.500019$ in eq ②

$$y^{(4)} = (6 - 0.49910 - 0.500019)\frac{1}{10}$$

$$= 0.50009$$

$\Rightarrow$ put $x = 0.49910$; $y = 0.50009$ in eq ③

$$z^{(4)} = (6 - 0.49910 - 0.50009)\frac{1}{10}$$

$$= 0.500081$$

$x^{(4)} = 0.49910$; $y^{(4)} = 0.50009$; $z^{(4)} = 0.500081$

V - Iteration

$\Rightarrow$ put $y = 0.50009$; $z = 0.500081$ in eq ①

$$x^{(5)} = (6 - 0.50009 - 0.500081)\frac{1}{10}$$

$$= 0.49910$$

$\Rightarrow$ put $x = 0.49910$; $z = 0.500081$ in eq ②

$$y^{(5)} = (6 - 0.49910 - 0.500081)\frac{1}{10}$$

$$= 0.500089$$

$\Rightarrow$ put $x = 0.49910$; $y = 0.500082$ in eq ③

$$z^{(5)} = (6 - 0.49910 - 0.500082)\frac{1}{10}$$

$$= 0.500082$$

Variable	1st	2nd	3rd	4th	5th
x	0.6	0.4974	0.4998	0.4999	0.4999
y	0.54	0.502	0.500014	0.5000	0.5000
z	0.486	0.50066	0.500019	0.5000	0.5000

3. Given Equation

$$\left.\begin{aligned} 8x_1 - 3x_2 + 2x_3 &= 20 \\ 4x_1 + 11x_2 - x_3 &= 33 \\ 6x_1 + 3x_2 + 12x_3 &= 36 \end{aligned}\right\} \rightarrow (A)$$

Equation (A) is a diagonally dominant system

$$8x_1 - 3x_2 + 2x_3 = 20$$

$$x_1 = (20 + 3x_2 - 2x_3)\frac{1}{8} \rightarrow (1)$$

$$4x_1 + 11x_2 - x_3 = 33$$

$$x_2 = (33 - 4x_1 + x_3)\frac{1}{11} \rightarrow (2)$$

$$6x_1 + 3x_2 + 12x_3 = 36$$

$$x_3 = (36 - 6x_1 - 3x_2)\frac{1}{12} \rightarrow (3)$$

I - Iteration

$\Rightarrow$ put $x_2 = 0$; $x_3 = 0$ in eq (1)

$$x_1^{(1)} = (20 + 3(0) - 2(0))\frac{1}{8}$$

$$= \frac{20}{8} = 2.5$$

$\Rightarrow$ put $x_1 = 2.5$; $x_3 = 0$ in eq (2)

$$x_2^{(1)} = (33 - 4(2.5) + 0)\frac{1}{11}$$

$$= 2.091$$

$\Rightarrow$ put $x_1 = 2.5$; $x_2 = 2.091$ in eq ③

$$x_3^{(1)} = (36 - 6(2.5) - 3(2.091))\frac{1}{12}$$

$$= 1.22725$$

$$= 1.23 \qquad x_1 = 2.5 ;\ x_2 = 2.091 ; x_3 = 1.23$$

II - Iteration

$\Rightarrow$ put $x_2 = 2.091$; $x_3 = 1.23$ in eq ①

$$x_1^{(2)} = [20 + 3(2.091) - 2(1.23)]\frac{1}{8}$$

$$= 2.976625$$

$$x_1 = 2.977$$

$\Rightarrow$ put $x_1 = 2.977$; $x_3 = 1.23$ in eq ②

$$x_2^{(2)} = (33 - 4(2.977) + 1.23)\frac{1}{11}$$

$$= 2.0293$$

$\Rightarrow$ put $x_1 = 2.977$; $x_2 = 2.0293$ in eq ③

$$x_3^{(2)} = [(36 - 6(2.977) - 3(2.0293)]\frac{1}{12}$$

$$= 1.004175$$

$$= 1.0042$$

$\therefore x_1^{(2)} = 2.977$; $x_2^{(2)} = 2.0293$; $x_3^{(2)} = 1.0042$

III - Iteration

$\Rightarrow$ put $x_2 = 2.0293$; $x_3 = 1.0042$ in eq ①

$$x_1^{(3)} = [20 + 3(2.0293) - 2(1.0042)]\frac{1}{8}$$

$$= 3.009$$

$\Rightarrow$ put $x_1 = 3.001$; $x_3 = 1.0042$ in eq ②

$$x_2^{(3)} = (33 - 4(3.001) + 1.0042)\frac{1}{11}$$

$$= 2.00008$$

$\Rightarrow$ put $x_1 = 3.001$; $x_2 = 2.000$ in eq ③

$$x_3^{(3)} = [(36 - 6(3.001) - 3(2.000)] \frac{1}{12}$$

$$= 0.9995$$

$\therefore x_1^{(3)} = 3.001$; $x_2^{(3)} = 2.000$; $x_3^{(3)} = 0.9995$

IV - Iteration

$\Rightarrow$ put $x_2 = 2.000$; $x_3 = 0.9995$ in eq ①

$$x_1^{(4)} = (20 + 3(2.000) \cancel{+0.9995} - 2(0.9995)) \frac{1}{8}$$

$$= 3.000$$

$\Rightarrow$ put $x_1 = 3.000$; $x_3 = 0.9995$ in eq ②

$$x_2^{(4)} = (33 - 4(3.000) + 0.9995) \frac{1}{11}$$

$$= 1.9990 = 2.000$$

$\Rightarrow$ put $x_1 = 3.000$; $x_2 = 1.9910$ in eq ③

$$x_3^{(4)} = (36 - 6(3.000) - 3(1.9910)] \frac{1}{12}$$

$$= 1.00225$$ $\therefore x_1^{(4)} = 3.000$; $x_2^{(4)} = 1.9910$ $= 2.000$

$x_3^{(4)} = 1.00225$

V - Iteration

$\Rightarrow$ put $x_2 = 1.9910$; $x_3 = 1.00225$ in eq ①

$$x_1^{(5)} = [20 + 3(1.9910) - 2(1.00225)] \frac{1}{8} = 3.000$$

$\Rightarrow$ put $x_1 = 3.000$; $x_3 = 1.00225$ in eq ②

$$x_2^{(5)} = (33 - 4(3.000) + 1.00225) \frac{1}{11} = 2.000$$

$\Rightarrow$ put $x_1 = 3.000$; $x_2 = 2.000$ in eq ③

$$x_3^{(5)} = [(36 - 6(3.000) - 3(2.000)] \frac{1}{12} = 1$$

$\therefore x_1^{(5)} = 3.000$; $x_2^{(5)} = 2.000$; $x_3^{(5)} = 1$

variable	1st	2nd	3rd	4th	5th
x	2.5	2.977	3.001	3.000	3.000
y	2.091	2.0293	2.000	2.000	2.000
z	1.23	1.0042	0.9995	1.00205	1.000

Date 14/12/2018

5. Solve $10x_1 - 2x_2 - x_3 - x_4 = 3$; $-2x_1 + 10x_2 - x_3 - x_4 = 15$; $-x_1 - x_2 + 10x_3 - 2x_4 = 15$; $-x_1 - x_2 - 2x_3 + 10x_4 = -9$ by Gauss - seidl method correct to three decimal places.

Sol) Given Equations

$$\left.\begin{array}{l} 10x_1 - 2x_2 - x_3 - x_4 = 3 \\ -2x_1 + 10x_2 - x_3 - x_4 = 15 \\ -x_1 - x_2 + 10x_3 - 2x_4 = 15 \\ -x_1 - x_2 - 2x_3 + 10x_4 = -9 \end{array}\right] \rightarrow (A)$$

Equation (A) is a diagonally dominant system

$$10x_1 - 2x_2 - x_3 - x_4 = 3$$

$$x_1 = \frac{1}{10}(3 + 2x_2 + x_3 + x_4) \longrightarrow (1)$$

$$-2x_1 + 10x_2 - x_3 - x_4 = 15$$

$$x_2 = (15 + 2x_1 + x_3 + x_4)\frac{1}{10} \rightarrow (2)$$

$$x_3 = \frac{1}{10}[15 + x_1 + x_2 + 2x_4] \rightarrow (3)$$

$$x_4 = \frac{1}{10}[-9 + x_1 + x_2 + 2x_3] \rightarrow (4)$$

I-Iteration

$\Rightarrow$ put $x_2 = 0$; $x_3 = 0$; $x_4 = 0$ in eq(1)

$$x_1 = \frac{1}{10}(3 + 2(0) + 0 + 0)$$

$$= 0.3$$

$\Rightarrow$ put $x_1 = 0.3$; $x_3 = 0$; $x_4 = 0$ in eq(2)

$$x_2 = \frac{1}{10}(15 + 2(0.3) + 0 + 0)$$

$$= 1.56$$

$\Rightarrow$ put $x_1 = 0.3$, $x_2 = 1.56$; $x_4 = 0$ in eq ③

$$x_3^{(1)} = \frac{1}{10}[15 + 0.3 + 1.56 + 0) = 1.686$$

$\Rightarrow$ put $x_1 = 0.3$; $x_2 = 1.56$, $x_3 = 1.686$ in eq ④

$$x_4^{(1)} = \frac{1}{10}(-9 + 0.3 + 1.56 + 2(1.686)]$$

$$= -0.377$$

$\therefore x_1^{(1)} = 0.3$; $x_2^{(1)} = 1.56$; $x_3^{(1)} = 1.686$; $x_4^{(1)} = -0.377$

II - Iteration

$\Rightarrow$ Put $x_2 = 1.56$; $x_3 = 1.686$; $x_4 = -0.377$ in eq ①

$$x_1^{(2)} = \frac{1}{10}(3 + 2(1.56) + 1.686 - 0.377)$$

$$= 0.74223$$

$\Rightarrow$ put $x_1 = 0.74223$; $x_3 = 1.686$; $x_4 = -0.377$ in eq ②

$$x_2^{(2)} = \frac{1}{10}(15 + 2(0.74223) + 1.686 - 0.377)$$

$$= 1.778695$$

$\Rightarrow$ put $x_1 = 0.74223$; $x_2 = 1.7795$; $x_4 = -0.377$ in eq ③

$$x_3^{(2)} = \frac{1}{10}(15 + 0.743 + 1.7795 + 2(-0.377))$$

$$= 1.6768$$

$\Rightarrow$ put $x_1 = 0.743$; $x_2 = 1.7795$; $x_3 = 1.6768$ in eq ④

$$x_4^{(2)} = \frac{1}{10}(-9 + 0.743 + 1.7795 + 2(1.6768))$$

$$= -0.31239$$

$x_1^{(2)} = 0.743$; $x_2^{(2)} = 1.779$; $x_3^{(2)} = 1.6768$; $x_4^{(2)} = -0.31239$

III - Iteration

$\Rightarrow$ put $x_2 = 1.779$; $x_3 = 1.6768$; $x_4 = -0.3124$ in eq①

$$x_1^{(3)} = \frac{1}{10}(3+2(1.779)+1.6768 + -0.3124)$$

$$= 0.7922$$

$\Rightarrow$ put $x_1 = 0.7922$; $x_3 = 1.6768$; $x_4 = -0.3124$ in eq ②

$$x_2^{(3)} = \frac{1}{10}(15+2(0.7922)+1.6768-0.3124)$$

$$= 1.79488 = 1.795$$

$\Rightarrow$ put $x_1 = 0.792$; $x_2 = 1.795$; $x_4 = -0.3124$ in eq ③

$$x_3^{(3)} = \frac{1}{10}[(15+0.792+1.795 \pm 2(0.3124))$$

$$= 1.696$$

$\Rightarrow$ put $x_1 = 0.792$; $x_2 = 1.795$; $x_3 = 1.696$ in eq ④

$$x_4^{(3)} = \frac{1}{10}[(-9+0.792+1.795+2(1.696)]$$

$$= -0.3021 = -0.302$$

$\therefore x_1^{(3)} = 0.792$; $x_2^{(3)} = 1.795$; $x_3^{(3)} = 1.696$; $x_4^{(3)} = -0.302$

IV - Iteration

$\Rightarrow$ put $x_2 = 1.795$; $x_3 = 1.696$; $x_4 = -0.302$ in eq ①

$$x_1^{(4)} = \frac{1}{10}(3+2(1.795)+1.696-0.302)$$

$$= 0.7984 = 0.798$$

$\Rightarrow$ put $x_1 = 0.798$; $x_3 = 1.696$; $x_4 = -0.302$ in eq ②

$$x_2^{(4)} = \frac{1}{10}(15+2(0.798)+1.696-0.302)$$

$$= 1.799$$

$\Rightarrow$ put $x_1 = 0.798$; $x_2 = 1.799$; $x_4 = -0.302$ in eq ③

$$x_3^{(4)} = \frac{1}{10}(15+0.798+1.799 \pm 2(0.302)]$$

$$= 1.6993 = 1.699$$

$\Rightarrow$ put $x_1 = 0.798$; $x_2 = 1.799$; $x_3 = 1.699$ in eq ④

$$x_4^{(4)} = \frac{1}{10}[-9 + 0.798 + 1.799 + 2(1.699)]$$

$$= -0.3005$$

$$= -0.300$$

$\therefore x_1^{(4)} = 0.798$; $x_2^{(4)} = 1.799$; $x_3^{(4)} = 1.699$; $x_4^{(4)} = -0.30$

V. Iteration

$\Rightarrow$ put $x_2 = 1.799$; $x_3 = 1.699$; $x_4 = -0.300$ in eq ①

$$x_1^{(5)} = \frac{1}{10}(3 + 2(1.799) + 1.699 - 0.300)$$

$$= 0.7997 = 0.799$$

$\Rightarrow$ put $x_1 = 0.798$; $x_3 = 1.699$; $x_4 = -0.300$ in eq ②

$$x_2^{(5)} = \frac{1}{10}(15 + 2(0.798) + 1.699 - 0.300)$$

$$= 1.7995 = 1.799$$

$\Rightarrow$ put $x_1 = 0.798$; $x_2 = 1.799$; $x_4 = -0.300$ in eq ③

$$x_3^{(5)} = \frac{1}{10}[15 + 0.798 + 1.799 + 2(0.300)]$$

$$= 1.6997 = 1.699$$

$\Rightarrow$ put $x_1 = 0.798$; $x_2 = 1.799$; $x_3 = 1.699$ in eq ④

$$x_4^{(5)} = \frac{1}{10}[-9 + 0.798 + 1.799 + 2(1.699)]$$

$$= -0.3005 = -0.300$$

variable	1st	2nd	3rd	4th	5th
x_1	0.3	0.743	0.792	0.798	0.799
x_2	1.56	1.779	1.795	1.799	1.799
x_3	1.686	1.6768	1.696	1.699	1.699
x_4	-0.377	-0.3124	-0.302	-0.300	-0.300

Gauss -

Solutions of Linear systems Direct Methods

1) Gaussian Elimination Method

This method of solving system of n linear equations in 'n' unknowns consists of eliminating the co-efficients in such a way that the system reduces to upper triangular system which may be solved by backward substitution.

1. Solve the Equations $2x+y+z=10; 3x+2y+3z=18; x+4y+9z=16$; by using Gauss elimination method.

Solu) Given Equations

$$\left.\begin{array}{l} 2x+y+z=10 \\ 3x+2y+3z=18 \\ x+4y+9z=16 \end{array}\right\} \rightarrow (1)$$

system (1) can be expressed in the form $AX=B$

where

$$A=\begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix};\ X=\begin{bmatrix} x \\ y \\ z \end{bmatrix};\ B=\begin{bmatrix} 10 \\ 18 \\ 16 \end{bmatrix}$$

Argumented matrix

$$[AB]=\begin{bmatrix} 2 & 1 & 1 & 10 \\ 3 & 2 & 3 & 18 \\ 1 & 4 & 9 & 16 \end{bmatrix}$$

$$\sim\begin{bmatrix} 2 & 1 & 1 & 10 \\ 0 & 1 & 3 & 6 \\ 0 & 7 & 17 & 22 \end{bmatrix} \begin{array}{l} R_2 \rightarrow 2R_2-3R_1 \\ R_3 \rightarrow 2R_3-R_1 \end{array}$$

70
21
$\overline{59}$

42
22
$\overline{20}$

$$\sim\begin{bmatrix} 2 & 1 & 1 & 10 \\ 0 & 1 & 3 & 6 \\ 0 & 0 & -4 & -20 \end{bmatrix} R_3 \rightarrow R_3-7R_2$$

which is a upper triangular matrix

$$2x+y+z=10;\ y+3z=6$$

$$-4z=-20$$

$$z=5$$

$$y+3(5)=6 \qquad ; \qquad 2x-9+5=10$$

$$y=6-15 \qquad 2x=14$$

$$y=-9 \qquad x=7;$$

$$x=7;\ y=-9;\ z=5$$

2. Solve $3x+y-z=3$; $2x-8y+z=-5$; $x-2y+9z=8$ by Gaussian elimination method

Given Equations

$$\left.\begin{array}{l} 3x+y-z=3 \\ 2x-8y+z=-5 \\ x-2y+9z=8 \end{array}\right] \rightarrow ①$$

system ① can be expressed in the form $AX=B$

$$A=\begin{bmatrix} 3 & 1 & -1 \\ 2 & -8 & 1 \\ 1 & -2 & 9 \end{bmatrix};\ X=\begin{bmatrix} x \\ y \\ z \end{bmatrix};\ B=\begin{bmatrix} 3 \\ -5 \\ 8 \end{bmatrix}$$

Argumented matrix

$$[AB]=\begin{bmatrix} 3 & 1 & -1 & 3 \\ 2 & -8 & 1 & -5 \\ 1 & -2 & 9 & 8 \end{bmatrix}$$

$$\sim\begin{bmatrix} 3 & 1 & -1 & 3 \\ 0 & -26 & 5 & -21 \\ 0 & -7 & 28 & 21 \end{bmatrix} \begin{array}{l} R_2 \rightarrow 3R_2-2R_1 \\ R_3 \rightarrow 3R_3-R_1 \end{array} \qquad \begin{array}{c} -24 \\ -2 \end{array}$$

$$\sim\begin{bmatrix} 3 & 1 & -1 & 3 \\ 0 & -26 & 5 & -21 \\ 0 & -1 & 4 & 3 \end{bmatrix} R_3 \rightarrow \frac{R_3}{7}$$

$$\sim \begin{bmatrix} 3 & 1 & -1 & 3 \\ 0 & -26 & 5 & -21 \\ 0 & 0 & 99 & 99 \end{bmatrix} R_3 \to 26R_3 + R_2$$

$\begin{array}{r} 1 \\ 26 \\ 3 \\ \hline 78 \\ 21 \\ \hline 99 \end{array}$ $\begin{array}{r} 2 \\ 26 \\ 4 \\ \hline 104 \\ 5 \\ \hline 99 \end{array}$

which is a upper triangular matrix

$3x + y - z = 3$

$-26y + 5z = -21$

$99z = 99$

$z = 1$

$-26y + 5 = -21$

$-26y = -21 - 5$

$-26y = -26$

$y = 1$

$3x + 1 - 1 = 3$

$x = 1$

154

$\therefore x = 1, y = 1, z = 1$

3. Solve $2x + y + z = 10$; $3x + 2y + 3z = 18$; $x + 4y + 9z = 16$ by using Gauss-Jordan-Method (only row operations)

Solu) Given Equations

$$\left.\begin{array}{l} 2x + y + z = 10 \\ 3x + 2y + 3z = 18 \\ x + 4y + 9z = 16 \end{array}\right\} \to (1)$$

system (1) can be expressed in the form $AX = B$ where

$$[A\ B] = \begin{bmatrix} 2 & 1 & 1 & 10 \\ 3 & 2 & 3 & 18 \\ 1 & 4 & 9 & 16 \end{bmatrix}$$

$$\sim \begin{bmatrix} 2 & 1 & 1 & 10 \\ 0 & 1 & 3 & 6 \\ 0 & 7 & 17 & 22 \end{bmatrix} \begin{array}{l} R_2 \to 2R_2 - 3R_1 \\ R_3 \to 2R_3 - R_1 \end{array}$$

$$\sim \begin{bmatrix} 2 & 1 & 1 & 10 \\ 0 & 1 & 3 & 6 \\ 0 & 0 & -4 & -20 \end{bmatrix} R_3 \to R_3 - 7R_2$$

$$\sim \begin{bmatrix} 2 & 1 & 1 & 10 \\ 0 & 1 & 3 & 6 \\ 0 & 0 & 1 & 5 \end{bmatrix} \quad R_3 \to R_3/-4$$

$$\sim \begin{bmatrix} 2 & 1 & 0 & 5 \\ 0 & 1 & 0 & -9 \\ 0 & 0 & 1 & 5 \end{bmatrix} \quad \begin{array}{l} R_1 \to R_1 - R_3 \\ R_2 \to R_2 - 3R_3 \end{array}$$

$$\sim \begin{bmatrix} 2 & 0 & 0 & 14 \\ 0 & 1 & 0 & -9 \\ 0 & 1 & 1 & 5 \end{bmatrix} \quad R_1 \to R_1 - R_2$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 7 \\ 0 & 1 & 0 & -9 \\ 0 & 0 & 1 & 5 \end{bmatrix} \quad R_1 \to R_1/2$$

$x = 7$; $y = -9$; $z = 5$

H.W.

4. Solve the equations $x+y+z=6$; $3x+3y+4z=20$; $2x+y+3z=13$; using partial pivoting Gaussian elimination method.

Solu) Given Equations

$$\left.\begin{array}{l} x+y+z=6 \\ 3x+3y+4z=20 \\ 2x+y+3z=13 \end{array}\right\} \to (1)$$

System (1) can be expressed in the form

$AX = B$ where

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 3 & 4 \\ 2 & 1 & 3 \end{bmatrix} ; X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} ; B = \begin{bmatrix} 6 \\ 20 \\ 13 \end{bmatrix}.$$

Argumented matrix

$$[AB] = \begin{bmatrix} 1 & 1 & 1 & 6 \\ 3 & 3 & 4 & 20 \\ 2 & 1 & 3 & 13 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 0 & 1 & 2 \\ 0 & -1 & 1 & 1 \end{bmatrix} \begin{array}{l} R_2 \to R_2 - 3R_1 \\ R_3 \to R_2 - 2R_1 \end{array}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & -1 & 1 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix} \quad R_2 \leftrightarrow R_3$$

which is a upper triangular matrix

$x+y+z=6$; $x+1+2=6$, $x=3$

$-y+z=1$; $-y+2=1$, $-y=-1$, $y=1$;

$z=2$

$\therefore x=3;\ y=1;\ z=2$

5. Solve the Equations $3x+y+2z=3$; $2x-3y-z=-3$; $x+2y+z=4$ by using Gauss Elimination method

Solu) Given Equations

$$\left.\begin{array}{l} 3x+y+2z=3 \\ 2x-3y-z=-3 \\ x+2y+z=4 \end{array}\right\} \to (1)$$

System (1) can be expressed in the form $AX=B$.

where $A = \begin{bmatrix} 3 & 1 & 2 \\ 2 & -3 & -1 \\ 1 & 2 & 1 \end{bmatrix}$; $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$; $B = \begin{bmatrix} 3 \\ -3 \\ 4 \end{bmatrix}$

Argumented matrix

$$[AB] = \begin{bmatrix} 3 & 1 & 2 & 3 \\ 2 & -3 & -1 & -3 \\ 1 & 2 & 1 & 4 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 4 \\ 2 & -3 & -1 & -3 \\ 3 & 1 & 2 & 3 \end{bmatrix} \quad R_1 \leftrightarrow R_3$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 4 \\ 0 & -7 & -3 & -11 \\ 0 & -5 & -1 & -9 \end{bmatrix} \begin{array}{l} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 3R_1 \end{array}$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 4 \\ 0 & -7 & -3 & -11 \\ 0 & 0 & 8 & -8 \end{bmatrix} \quad R_3 \to 7R_3 - 5R_2.$$

which is an upper triangular matrix.

$x+2(2)-1=4$; $x+2y+z=4$

$x+4-1=4$ $\quad -7y-3z=-11$ $\quad ; -7y-3(-1)=-11$

$x=1$ $\quad 8z=-8$ $\quad -7y+3=-11$

$z=-1$ $\quad -7y=-14$

$y=2$

$\therefore x=1;\ y=2;\ z=-1$

6. Solve the Equations $10x+y+z=12$; $2x+10y+z=13$ and $x+y+5z=7$ by Gauss - Jordan Method

Solu) Given Equations

$$\left.\begin{aligned} 10x+y+z&=12 \\ 2x+10y+z&=13 \\ x+y+5z&=7 \end{aligned}\right\} \to (1)$$

system (1) can be expressed in the form

$AX=B$

$$A=\begin{bmatrix} 10 & 1 & 1 \\ 2 & 10 & 1 \\ 1 & 1 & 5 \end{bmatrix};\ X=\begin{bmatrix} x \\ y \\ z \end{bmatrix};\ B=\begin{bmatrix} 12 \\ 13 \\ 7 \end{bmatrix}$$

Argumented matrix

$$[AB]=\begin{bmatrix} 10 & 1 & 1 & 12 \\ 2 & 10 & 1 & 13 \\ 1 & 1 & 5 & 7 \end{bmatrix}$$

$$\sim \begin{bmatrix} 10 & 1 & 1 & 12 \\ 0 & 49 & 4 & 53 \\ 0 & 9 & 49 & 58 \end{bmatrix} \quad \begin{aligned} R_2 &\to 5R_2 - R_1 \\ R_3 &\to 10R_3 - R_1 \end{aligned}$$

$$\sim \begin{bmatrix} 10 & 1 & 1 & 1 \\ 0 & 49 & 4 & 53 \\ 0 & 0 & 2365 & 2365 \end{bmatrix} \quad R_3 \to 49R_3 - 9R_2$$

$$\sim \begin{bmatrix} 20 & 1 & 1 & 1 \\ \cdot & & & \end{bmatrix} \quad R_2 \leftrightarrow$$

$441 \quad \frac{65}{12} \quad 53$

$2401 \quad \frac{70}{12} \quad 58$

$$\sim \begin{bmatrix} 1 & 1 & 5 & 7 \\ 2 & 10 & 1 & 13 \\ 10 & 1 & 1 & 12 \end{bmatrix} R_1 \leftrightarrow R_3$$

$$\sim \begin{bmatrix} 1 & 1 & 5 & 7 \\ 0 & 8 & -9 & -1 \\ 0 & -9 & -49 & -58 \end{bmatrix} \begin{matrix} R_2 \leftrightarrow R_2 - 2R_1 \\ R_3 \rightarrow R_3 - 10R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & -8 & -44 & -51 \\ 0 & 8 & -9 & -1 \\ 0 & -9 & -49 & -58 \end{bmatrix} R_1 \rightarrow R_1 + R_3$$

$$\sim \begin{bmatrix} -1 & +8 & +44 & +51 \\ 0 & 8 & -9 & -1 \\ 0 & 9 & 49 & 58 \end{bmatrix} \begin{matrix} R_1 \rightarrow \frac{R_1}{-1} \\ R_3 \rightarrow \frac{R_3}{-1} \end{matrix}$$

392
464

$$\sim \begin{bmatrix} -1 & 8 & 44 & 51 \\ 0 & 8 & -9 & -1 \\ 0 & 0 & 473 & 473 \end{bmatrix} R_3 \rightarrow 8R_3 - 9R_2$$

$$\sim \begin{bmatrix} -1 & 8 & 44 & 51 \\ 0 & 8 & -9 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix} R_3 \rightarrow \frac{R_3}{473}$$

$$\sim \begin{bmatrix} -1 & 0 & 53 & 52 \\ 0 & 8 & 0 & 8 \\ 0 & 0 & 1 & 1 \end{bmatrix} \begin{matrix} R_1 \rightarrow R_1 - R_2 \\ R_2 \rightarrow R_2 + 9R_3 \end{matrix}$$

$$\sim \begin{bmatrix} -1 & 0 & 53 & 52 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} R_2 \rightarrow \frac{R_2}{8}$$

$$\sim \begin{bmatrix} +1 & 0 & 0 & +1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} R_1 \rightarrow \frac{R_1 - 53R_3}{-1}$$

$\therefore x = 1;\ y = 1;\ z = 1$

7. Solve the Equations

$10x_1 + x_2 + x_3 = 12$; $x_1 + 10x_2 - x_3 = 10$ and $x_1 - 2x_2 + 10x_3 = 9$ by Gauss - Jordan method

Sol: Given Equations

$$\left.\begin{aligned} 10x_1 + x_2 + x_3 &= 12 \\ x_1 + 10x_2 - x_3 &= 10 \\ x_1 - 2x_2 + 10x_3 &= 9 \end{aligned}\right\} \rightarrow (1)$$

System (1) can be expressed in the form $AX = B$

$$A = \begin{bmatrix} 10 & 1 & 1 \\ 1 & 10 & -1 \\ 1 & -2 & 10 \end{bmatrix} ; \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} ; \quad B = \begin{bmatrix} 12 \\ 10 \\ 9 \end{bmatrix}$$

Argumented matrix

$$[AB] = \begin{bmatrix} 10 & 1 & 1 & 12 \\ 1 & 10 & -1 & 10 \\ 1 & -2 & 10 & 9 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -2 & 10 & 9 \\ 1 & 10 & -1 & 10 \\ 10 & 1 & 1 & 12 \end{bmatrix} \quad R_1 \leftrightarrow R_3$$

$$\sim \begin{bmatrix} 1 & -2 & 10 & 9 \\ 0 & 12 & -11 & 1 \\ 0 & 21 & -99 & -78 \end{bmatrix} \quad \begin{aligned} R_2 &\rightarrow R_2 - R_1 \\ R_3 &\rightarrow R_3 - 10R_1 \end{aligned}$$

$$\sim \begin{bmatrix} 1 & -2 & 10 & 9 \\ 0 & 12 & -11 & 1 \\ 0 & 7 & -33 & -26 \end{bmatrix} \quad R_3 \rightarrow \frac{R_3}{3}$$

$$\sim \begin{bmatrix} 1 & -2 & 10 & 9 \\ 0 & 12 & -11 & 1 \\ 0 & 0 & -319 & -319 \end{bmatrix} \quad R_3 \rightarrow 12R_3 - 7R_2$$

$$\sim \begin{bmatrix} 1 & -2 & 10 & 9 \\ 0 & 12 & -11 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \quad R_3 \rightarrow \frac{R_3}{-319}$$

$$\sim \begin{bmatrix} 1 & -2 & 10 & 9 \\ 0 & 12 & 0 & 12 \\ 0 & 0 & 1 & 1 \end{bmatrix} \quad R_2 \rightarrow R_2 + 11R_3$$

$$\sim \begin{bmatrix} 1 & -2 & 10 & 9 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 9 \end{bmatrix} R_2 \to \frac{R_2}{12}$$

$$\sim \begin{bmatrix} 1 & 0 & 10 & 21 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} R_1 \to R_1 + 2R_2$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} R_1 \to R_1 - 10R_3$$

$x_1 = 2$; $x_2 = 1$; $x_3 = 1$

8. Solve the system of Equations by Gauss-Seidel method $20x + y - 2z = 17$; $3x + 20y - z = -18$; $2x - 3y + 20z = 25$

Solu) Given Equations

$$\left.\begin{aligned} 20x + y - 2z &= 17 \\ 3x + 20y - z &= -18 \\ 2x - 3y + 20z &= 25 \end{aligned}\right\} \to (A)$$

Equation (A) can be expressed [in the form $AX = B$

where $A = \begin{bmatrix} 20 & 1 & -2 \\ 3 & 20 & -1 \\ 2 & -3 & 20 \end{bmatrix}$; $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$; $B = \begin{bmatrix} 17 \\ -18 \\ 25 \end{bmatrix}$

Arg] equation (A) is a diagonally dominant

$$x = (17 - y + 2z)\frac{1}{20} \to (1)$$

$$y = (-18 - 3x + z)\frac{1}{20} \to (2)$$

$$z = (25 - 2x + 3y)\frac{1}{20} \to (3)$$

I- Iteration

$\Rightarrow$ put $y = 0$; $z = 0$ in eq (1)

$$x^{(1)} = (17 - 0 + 2(0))\frac{1}{20} = 0.85$$

$\Rightarrow$ put $x = 0.85$; $z = 0$ in eq ②

$$y^{(1)} = (-18 - 3(0.85) + 0)\frac{1}{20}$$

$$= -1.0275$$

$\Rightarrow$ put $x = 0.85$; $y = -1.0275$ in eq ③

$$z^{(1)} = (25 - 2(0.85) + 3(-1.0275))\frac{1}{20}$$

$$= 1.010875$$

$$= 1.0109$$

$x^{(1)} = 0.85$; $y^{(1)} = -1.0275$; $z^{(1)} = 1.0109$

II - Iteration

$\Rightarrow$ put $y = -1.0275$; $z = 1.0109$ in eq ①

$$x^{(2)} = (17 + 1.0275 + 2(1.0109))\frac{1}{20}$$

$$= 1.002465$$

$$= 1.0025$$

$\Rightarrow$ put $x = 1.0025$; $z = 1.0109$ in eq ②

$$y^{(2)} = (-18 - 3(1.0025) + 1.0109)\frac{1}{20}$$

$$= -0.99983$$

$$y = -0.9998 = -0.9910$$

$\Rightarrow$ put $x = 1.0025$; $y = -0.9910$ in eq ③

$$z^{(2)} = [(25 - 2(1.0025) + 3(0.9910)]\frac{1}{20}$$

$$= 1.0011$$

$\therefore x^{(2)} = 1.0025$; $y^{(2)} = -0.9928$; $z^{(2)} = 1.0011$

III - Iteration

$\Rightarrow$ put $y = -0.9910$; $z = 1.0011$ in eq ①

$$x^{(3)} = (17 + 0.9910 + 2(1.0011))\frac{1}{20}$$

$= 0.99966$

$= 0.999 = 1.00$

$\Rightarrow$ put $x=1$; $z = 1.0011$ in eq ②

$$y^{(3)} = (-18 - 3(1) + 1.0011)\frac{1}{20}$$

$$= -0.999945$$

$$= -1.000$$

$\Rightarrow$ put $x = 1$; $y = -1$ in eq ③

$$z^{(3)} = (25 - 2(1) - 3(1))\frac{1}{20}$$

$$= 1$$

$\therefore x^{(3)} = 1$; $y^{(3)} = -1$; $z^{(3)} = 1$

IV - Iteration

$\Rightarrow$ put $y = 1$; $z = 1$ in eq ①

$$x^{(4)} = (17 + 1 + 2(1))\frac{1}{20}$$

$$= 0.99 = 1$$

$\Rightarrow$ put $x = 1$; $z = 1$ in eq ②

$$y^{(4)} = (-18 - 3(1) + 1)\frac{1}{20}$$

$$= -1$$

$\Rightarrow$ put $x = 1$; $y = -1$ in eq ③

$$z^{(4)} = (25 - 2(1) - 3)\frac{1}{20}$$

$$= 1$$

$\therefore x^{(4)} = 1$; $y^{(4)} = -1$; $z^{(4)} = 1$

Variable	1st	2nd	3rd	4th
x	0.815	1.0025	1	1
y	-1.0275	-0.9910	-1	-1
z	1.0109	1.0011	1	1

9. Solve the following system of equations by using Gauss - Seidel method correct to three decimal places. $8x-3y+2z=20$; $4x+11y-z=33$; $6x+3y+12z=35$

Solu) Given Equations

$$\left.\begin{aligned} 8x-3y+2z &= 20 \\ 4x+11y-z &= 33 \\ 6x+3y+12z &= 35 \end{aligned}\right\} \rightarrow (A)$$

system (A) is a diagonally dominant system where

$$x = \frac{1}{8}(20+3y-2z) \rightarrow (1)$$

$$y = \frac{1}{11}(33-4x+z) \rightarrow (2)$$

$$z = \frac{1}{12}(35-6x-3y) \rightarrow (3)$$

I - Iteration

$\Rightarrow$ put $y=0$; $z=0$ in eq (1)

$$x^{(1)} = \frac{1}{8}(20+3(0)-2(0))$$

$$= 2.5$$

$\Rightarrow$ put $x=2.5$; $z=0$ in eq (2)

$$y^{(1)} = \frac{1}{11}(33-4(2.5)+0) = 2.09090$$

$$= 2.091$$

$\Rightarrow$ put $x=2.5$; $y=2.091$ in eq (3)

$$z^{(1)} = \frac{1}{12}(35-6(2.5)-3(2.091))$$

$$= 1.14439166 = 1.1444$$

$\therefore x^{(1)} = 2.5$; $y^{(1)} = 2.091$; $z^{(1)} = 1.444$

II- Iteration

$\Rightarrow$ put $(x = 2.5)$ $y = 2.091$; $z = 1.444$ in eq ①

$$x^{(2)} = \frac{1}{8}(20 + 3(2.091) - 2(1.444))$$

$$= 2.923125$$

$$= 2.923$$

$\Rightarrow$ put $x = 2.923$; $z = 1.444$ in eq ②

$$y^{(2)} = \frac{1}{11}(33 - 4(2.923) + 1.444)$$

$$= 2.0683636$$

$$= 2.068$$

$\Rightarrow$ put $x = 2.923$; $y = 2.068$ in eq ③

$$z^{(2)} = \frac{1}{12}(35 - 6(2.923) - 3(2.068))$$

$$= 0.938166$$

$$= 0.938$$

$\therefore$ $x^{(2)} = 2.923$; $y^{(2)} = 2.068$; $z^{(2)} = 0.938$

III- Iteration

$\Rightarrow$ put $y = 2.068$; $z = 0.938$ in eq ①

$$x^{(3)} = \frac{1}{8}(20 + 3(2.068) - 2(0.938))$$

$$= 3.041$$

$\Rightarrow$ put $x = 3.041$; $z = 0.938$ in eq ②

$$y^{(3)} = \frac{1}{11}(33 - 4(3.041) + 0.938)$$

$$= 1.9794545 = 1.979$$

$\Rightarrow$ put $x = 3.041$; $y = 1.979$ in eq ③

$$z^{(3)} = \frac{1}{12}(35 - 6(3.041) - 3(1.979))$$

$$= 0.901166 = 0.901$$

$\therefore x^{(3)} = 3.041 ; y^{(3)} = 1.979 ; z^{(3)} = 0.901$

IV - Iteration

$\Rightarrow$ put $y = 1.979 ; z = 0.901$ in eq ①

$$x^{(4)} = \frac{1}{8}(20 + 3(1.979) - 2(0.901))$$

$$= 3.016875$$

$$= 3.017$$

$\Rightarrow$ put $x = 3.017 ; z = 0.901$ in eq ②

$$y^{(4)} = \frac{1}{11}(33 - 4(3.017) + 0.901)$$

$$= 1.984818$$

$$= 1.985$$

$\Rightarrow$ put $x = 3.017 ; y = 1.985$ in eq ③

$$z^{(4)} = \frac{1}{12}(35 - 6(3.017) - 3(1.985))$$

$$= 0.9119166$$

$$= 0.912$$

$x^{(4)} = 3.017 ; y^{(4)} = 1.985 ; z^{(4)} = 0.912$

V - Iteration

$\Rightarrow$ put $y = 1.985 ; z = 0.912$ in eq ①

$$x^{(5)} = \frac{1}{8}(20 + 3(1.985) - 2(0.912))$$

$$= 3.016375 = 3.016$$

$\Rightarrow$ put $x = 3.016 ; z = 0.912$ in eq ②

$$y^{(5)} = \frac{1}{11}(33 - 4(3.016) + 0.912)$$

$$= 1.9861818$$

$$= 1.986$$

$\Rightarrow$ put $x = 3.016$; $y = 1.986$; in eq ③

$$z^{(5)} = \frac{1}{12}(35 - 6(3.016) - 3(1.986))$$
$$= 0.9121666$$
$$= 0.912$$

$\therefore x^{(5)} = 3.016$; $y^{(5)} = 1.986$; $z^{(5)} = 0.912$

VI - Iteration

$\Rightarrow$ put $y = 1.986$; $z = 0.912$ in eq ①

$$x^{(6)} = \frac{1}{8}(20 + 3(1.986) - 2(0.912))$$
$$= 3.01675 = 3.016$$

$\Rightarrow$ put $x = 3.016$; $z = 0.912$ in eq ②

$$y^{(6)} = \frac{1}{11}(33 - 4(3.016) - 7(0.912))$$
$$= \cancel{1.654545}\ 1.98618$$
$$= \cancel{1.655}\ 1.986$$

$\Rightarrow$ put $x = 3.016$; $y = 1.986$ in eq ③

$$z^{(6)} = \frac{1}{12}(35 - 6(3.016) - 3(1.986))$$
$$= 0.9121666$$
$$= 0.912$$

$x^{(6)} = 3.016$; $y^{(6)} = 1.986$; $z^{(6)} = 0.912$

Variable	I	II	III	IV	V	VI
x	2.5	2.923	3.041	3.017	3.016	3.016
y	2.091	2.068	1.979	1.985	1.986	1.986
z	1.444	0.938	0.901	0.912	0.912	0.912